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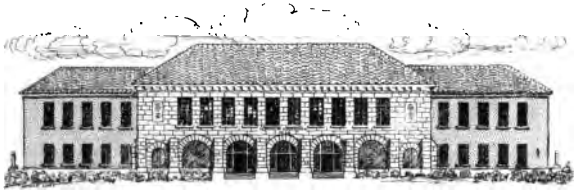
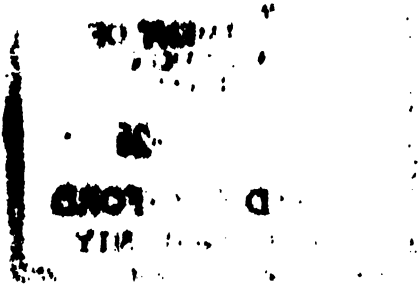
CHOOL
MATHEMATICS

HART

BOOK ONE





WELLS AND HART MATHEMATICAL SERIES

JUNIOR HIGH SCHOOL MATHEMATICS

BY

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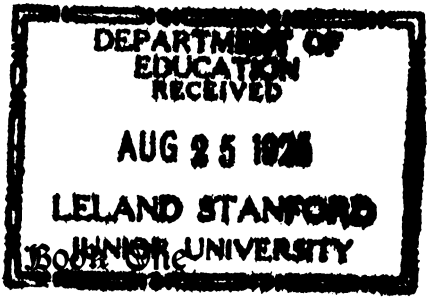

Book One

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This book may be had
with answers or *without answers*
at the same price.

Answer books, bound in paper, may be
obtained free of charge by *teachers*.

PRINTED IN U. S. A.

PREFACE

THIS book is the first of a series presenting a reorganized course in mathematics. It is designed for grade seven or the lowest junior high school year. The following principles have been observed in the preparation of it.

I. **The course must be practical.** In grade seven, this means that:

1. *There must be continued drill upon the fundamentals.*

In this text, the first forty-seven pages contain a systematic review of the fundamental operations. No short cuts or special devices are taught because most of the pupils of this grade are too inexpert with ordinary methods of computation to justify encouraging them to use special devices. The review is made systematic so that the teacher can easily assign work for the whole class or for individuals of the class without "dictating examples," and so that the pupils themselves can refer to it for self-instruction whenever necessary. On pages 211–219, are abstract drill exercises which can be used as suggested on page 211. This material and the miscellaneous review problems on pages 201–210 will be found particularly useful while the class is studying the geometry of Chapters VIII–XIII.

2. *The instruction on new topics must be limited to processes and applications encountered by the average person.*

By thus rigidly limiting the instruction to the common problems and processes, these are taught more thoroughly and time is gained in which to accomplish the other purposes of the course.

In this text, Chapters IV and V are devoted in an unusual manner to "finding the percentage" and its applications. These are the most common percentage problems. The discount problems in

Chapter V involve only one discount, — the only kind most people ever have to solve. "Finding the rate" and "finding the base" are taught in Chapter VI, with deserved emphasis upon the former. "Finding the interest" on a sum of money by the ordinary general method is the sole subject of instruction in Chapter VII, — the six per cent method, a special process, and the problems of finding the rate, the time, or the principal being postponed until grade eight.

II. The course must be enriched. By the time pupils reach this grade, a new appeal must be made to them to gain their interest, because they have become somewhat bored with the computational arithmetic. This cannot be done sufficiently by the use of problem material, local or otherwise, grouped about some central theme, as is so often attempted. On the other hand, the beginnings of literal arithmetic and inductive geometry will accomplish this purpose.

In this text, the formula is introduced in connection with percentage (pp. 61, 80, 94) and the areas of the plane figures studied in Chapters VII–XIII. This use of the formula is part of a thoroughly considered plan which is developed in subsequent books; it should not be considered lightly, as the use of formulæ is now admitted to be one of the central topics for instruction in algebra.

Particular attention, however, is directed to the geometry given in Chapters VIII–XIII. The instruction in this text is centered around the mensuration facts connected with the common plane figures and the rectangular parallelopiped, — these being the practical parts of geometry for most people. More than usual attention has been given to developing the geometrical concepts nvolved by drawing, measuring, and other informal exercises. Experience in teaching this material to children of this grade proves that they can master it in its present form and that they greatly enjoy doing so.

III. The course must be thorough. Standardized tests disclose the need of more thorough instruction.

In this text, a special effort has been made to instruct in a thorough manner. Observe the separation of the subject matter into small teaching units (*e.g.* §§ 24, 26–28; 29–30; 31). Observe that the

instruction is given in simple language and quite fully (*e.g.* §§ 7, 34, 35, 36, 37, 43). Observe the large number of examples and problems provided. Separation into groups of oral and written examples has been avoided because pupils should form the habit of doing computations mentally whenever possible. Observe that the instruction is topical, but that review lists of miscellaneous character are given from time to time (*e.g.* pp. 28–29, 44–47, 63–65, 77–79).

IV. The course must begin the process of rationalizing mathematics. In grades I–VI, processes and skills are emphasized, — "how" rather than "why". operations are performed in a certain manner. One of the major aims of secondary mathematics (grades VII–XII) is that of rationalizing mathematical processes. A strong argument may be made for the proposition that such rationalizing in the junior high school shall be chiefly inductive and, in the senior high school, chiefly deductive.

In this text, a start is made with the inductive rationalizing of certain processes with which the pupils are already familiar. (§§ 7, 10, 12, 16, 17.) Chiefly and most effectively, however, this form of thinking is developed in connection with the geometry. (See Ex. 5, p. 126; Ex. 11, p. 128; Ex. 1, p. 136; Ex. 3, p. 137; Ex. 5, p. 139; § 65; § 66; Ex. 1, p. 187.) It is urged that these geometry lessons be "tried out" as printed; if necessary, supply more questions or exercises of the same sort, but as far as possible avoid omitting any of them or taking them in other order. The educational value of this geometry depends more upon how it is taught than upon the extent of the instruction.

V. The course must prepare for later courses. The mathematics of grades seven and eight have an important rôle in the process of linking together the elementary school and the senior high school. The gap between the mathematics of these two schools has been notorious. This gap, moreover, will not be bridged by merely placing some of the secondary mathematics in these grades seven

and eight; to do much of algebra or geometry there is merely jumping the gap and robbing the later courses. The articulation will be satisfactory only if the pupils acquire in the seventh and eighth grades certain correct ideas and mathematical habits and an interest in mathematics which will enable them to study formal algebra and geometry successfully. The articulating process may be started in grade seven with the study of the formula and of inductive geometry.

In this text, the formula is taught in §§ 32 and 33, and with it the, for pupils, difficult idea of "substituting" in a formula. This, used repeatedly thereafter, is the only algebraic idea taught in this book. In the chapters on geometry, the definitions and point of view are consistent with the best practices in high school geometry. Besides the more obvious instances of preparation for high school geometry, attention is directed to the psychological preparation for the "superposition" process which is a result of the frequent use of tracing paper. (See Ex. 2, p. 115; Ex. 6, p. 123; Ex. 1, p. 128; Ex. 5, p. 139; Ex. 1, § 66, p. 156.)

VI. The course must be flexible. All schools do not wish and cannot use the same material.

Enough material has been provided for a full year of work. Certain sections, marked supplementary, are recommended for omission if it becomes necessary to save time somewhere. These sections will be treated adequately in subsequent courses in mathematics. (See §§ 67–72, and Chapter XII.) Some or all of these may be omitted if more time is required for the rest of the book.

Chapters I–VII constitute approximately one half year of work. If they are studied during the first semester, review examples from these chapters and from pages 201–210 should be studied during the second semester, while studying the geometry of Chapters VIII–XIII.

In some schools, especially such as have semi-annual promotions, it may be desired to have some of the geometry in the first semester and some of the percentage and interest problems in the second semester. In this case, the following plan is suggested:

First semester: Chapters I–V, VIII, IX, and reviews.

Second semester: Chapters VI, X, VII, XI–XIII, and reviews.

Suggestions to the Teacher

1. Have all the pupils use the same kind of paper and a No. 2 lead pencil.

2. Obtain a pencil sharpener and attach it to the wall near the door by which the pupils enter the room.

3. Insist upon neat and orderly work from the pupils.

4. When doing the geometry work, the pupils need to be cautioned repeatedly about doing their work neatly and accurately. They cannot do satisfactory work without a sharp pencil and tools. A combination protractor, square, and rule has been furnished with the text. Since it is not needed until the work in geometry begins, it may be advisable to collect all the cards for safe keeping until then. The cards furnished are made of as durable cardboard as can be obtained. If the one furnished is lost or rendered unfit for use, a new one of the same or similar sort should be gotten by the pupil.

Pupils of this grade are surprisingly inaccurate in measuring and inexpert in using tools. The only remedy is individual attention. A teacher can give such attention and direction while passing up and down the aisles, when giving directions during a drawing lesson or when supervising a "supervised study" lesson.

Note. The combination protractor furnished with this text may be bought from D. C. Heath & Co. at the following rates:

Less than twenty-five at 5 cents net each.
Twenty-five or more at 4 cents each.

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The numbers refer to pages

JUNIOR HIGH SCHOOL MATHEMATICS

BOOK I

I. BRIEF GENERAL REVIEW

1. The figures 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 are called **digits.** By means of them, the **integers** or whole numbers, the **common fractions,** and the **decimal fractions** are written.

A number like 5,462 should be read *five thousand, four hundred, sixty-two,* not five thousand *and* four hundred *and* sixty-two.

The number 265.078 should be read *two hundred sixty-five and seventy-eight thousandths.*

EXERCISE 1

1. Name all the digits.

2. Tell what each digit of the number 5,372 represents.

3. Read: 3,749; 5,630; 7,204; 9,063; 4,007.

4. Read: 30,248; 725,692; 800,405.

5. Read: 8.1; 785.25; 37.234; .8695; 23.046.

6. Read: $\frac{23}{81}$; $\frac{5}{21}$; $\frac{13}{32}$; $\frac{19}{64}$; $\frac{7}{18}$.

7. In the fraction $\frac{23}{64}$, what name is given to the number 23? to the number 64?

8. When two numbers are added, the numbers added are called **addends.** What name is given to the result?

9. If 39 is subtracted from 50, what is the result? What name is given to the result? to the number 39? to the number 50?

10. If 24 is multiplied by 16, what is the result? What name is given to the result? to the number 24? to the number 16?

11. If 27 is divided by 6, what is the quotient? What is the remainder? What name is given to the number 27? to the number 6?

12. To each of the following numbers, add 6. Be accurate. Do not hurry. Speed will come with practice.

19 44 73 65 32 27 48 96

NOTE. — Similarly add 8, or 5, or 9, or 7 to each of these numbers. Any pupil who is inaccurate or slower than the majority of the class should practice on other examples of the same sort until greater speed and accuracy have been attained. Additional drill examples are provided on p. 211.

13. (*Oral drill.*) Find the sums of the following columns:

a	*b*	*c*	*d*	*e*	*f*	*g*	*h*	*i*	*j*	*k*	*l*
8	6	2	9	5	7	6	4	9	3	5	7
5	7	8	7	9	5	2	3	8	7	9	6
9	4	5	6	2	7	6	8	9	4	6	3
7	3	5	4	7	8	9	6	5	3	8	4
6	5	8	4	7	5	3	9	7	5	4	8
4	7	5	8	2	4	7	6	9	7	4	6
5	8	7	4	6	9	7	8	3	5	6	7

NOTE. — If these exercises are used for written drill, time can be saved by not having the numbers copied from the text; have only the sums written.

14. Find the sums of the following columns:

b	*c*	*d*	*e*	*f*	*g*	*h*	*i*	*j*
53	64	37	98	43	65	27	92	39
78	98	56	29	27	93	54	68	54
84	75	82	78	69	43	37	56	85
60	54	19	74	92	58	39	27	46
39	46	58	83	74	95	62	39	48

15. Keep a record of the number of minutes that it kes you to get the sums of the following:

	b	*c*	*d*	*e*	*f*	*g*	*h*
6	386	674	69	751.	2.65	3.78	35.84
3	764	546	483	29.4	53.6	62.8	316.7
8	905	863	286	684.3	46.89	385.29	45.72
4	869	273	658	405.04	57.83	475.78	354.06
6	869	247	87	567.7	45.86	35.67	263.17

16. Find the sums of the following columns. If you e inaccurate, make up other columns like these and find eir sums:

	b	*c*	*d*	*e*	*f*	*g*	*h*
7	866	672	709	29.5	203.74	9.673	13.45
8	743	8463	56	624.3	23.6	84.009	963.05
4	280	357	426	8.22	7.86	2.73	537.42
1	599	432	85	96.54	537.24	68.72	593.47
4	729	3687	258	86.29	364.26	57.91	27.35
6	645	5832	274	428.94	17.05	5.689	58.26
0	294	179	687	374.20	417.48	5.27	537.80
8	307	782	157	239.51	361.25	48.198	246.75

2. When **copying figures**, before adding them or subcting them, take time to write them neatly in columns, eping the decimal points in a column. After adding olumn upward, add it again downward in order to test e correctness of your first sum.

EXERCISE 2

1. Copy and find the sum of the following amounts:
$78.32, $806.94, $1,482.27, $22.93, $475.64, $1,239.52, $325.68.

2. A farmer, testing one of his cows, found that the weights of milk produced by the cow during one week were: Monday, 37 lb.; Tuesday, 36 lb.; Wednesday, 39 lb.; Thursday, 38 lb.; Friday, 37.5 lb.; Saturday, 39 lb.; and Sunday, 38.25 lb. What was the total amount of milk produced by that cow during the week?

3. A grocer's receipts for a week were: Monday, $132.45; Tuesday, $93.62; Wednesday, $104.75; Thursday, $125.57; Friday, $89.74; Saturday, $178.96. What were his total receipts for the week?

4. A family, planning a new home, received the following bids from contractors: carpenter, $3022.70; mason, $1465; plasterer, $727.50; tinsmith, $132.45; plumber, $825.60; heating contractor, $682.75; electrician, $195.43; hardware dealer, $211.50. What is the total of these bids?

5. Farmer Davies delivered at the milk condensery the following amounts of milk one week: Monday, 275 lb.; Tuesday, 283 lb.; Wednesday, 279 lb.; Thursday, 300 lb.; Friday, 290 lb.; Saturday, 285 lb.; Sunday, 291 lb. How much milk did he deliver during the week?

6. John's father asked him to determine the total of the following expenditures during the month of January: coal, $115.75; taxes, $67.50; house expenses, $95.32; club dues, $35.00; insurance, $78.93; savings account, $20.00. What was the total?

7. The treasurer of a village fire and lightning insurance company reported the following receipts for the year:

cash on hand on January 1st, $3,962.36; received from Mr. T. $2,049.81; from Mr. L. $1,607.19; from Mr. B. $1,382.34; from Mr. F. $1,342.49; from Mr. W. $849.24; from Mr. T. $812.33; from Mr. H. $788.47; from Mr. C. $149.83. What were the total receipts for the year?

EXERCISE 3

Addition, subtraction, and multiplication drill

1. Find the sums of the following columns:

a	*b*	*c*	*d*	*e*	*f*	*g*
332	587	670	894	47.6	29.604	357.85
420	295	873	767	285.03	3.27	45.86
375	238	436	574	9.47	757.28	621.085
645	327	766	861	62.9	6.542	37.004
294	543	389	763	205.51	83.375	159.04
683	695	374	517	372.69	342.065	47.832
946	507	638	792	678.08	425.63	128.756
542	964	877	469	53.16	43.985	138.206

2. From each of the following numbers subtract 5, writing down only the remainder, or giving the result orally:

78 93 64 82 45 36 29 57

For oral drill, subtract also 7, 9, 8, 6, etc.

3. Subtract the lower number from the upper number in the following:

a	*b*	*c*	*d*	*e*	*f*	*g*
87354	59270	73605	543.76	76.294	683.49	351.9
81926	43947	67826	276.95	5.386	273.08	45.782

4. Multiply each of the following numbers by 4, writing down only the products, or giving the results orally:

18 27 65 34 49 76 92 53

For oral drill, or written drill, multiply the numbers in Example 4 by 5, 7, 6, 8, and 9.

5. Find the following products:

a	*b*	*c*	*d*	*e*	*f*	*g*	*h*
643	564	945	852	967	375	609	804
82	76	368	706	450	206	950	579

6. Keep a record of the time it takes you to obtain the following products:

a	*b*	*c*	*d*	*e*	*f*	*g*	*h*
659	468	275	367	927	436	684	759
64	47	38	92	75	49	73	87

NOTE FOR PUPIL. — If you find that you are much slower than the majority of the class, make up ten examples like these, solve them, and bring them to your teacher.

3. Keeping accounts is necessary in business and is a good practice in families, as a means of systematizing expenditures.

A BOY'S GARDEN ACCOUNT

1921	EXPENDITURES			1921	RECEIPTS		
Apr. 18	Plowing 1½ acres	$ 3	00	Sept. 29	30 bu. corn @ $1.65	$49	50
19	Harrowing and marking	1	50	Oct. 5	11 bu. potatoes @ $1.50	16	50
24	Seed corn for ¾ acre	1	25		Total	66	00
25	Planting corn		75				
May 2	Seed potatoes for ½ acre	5	00				
3	Planting potatoes	1	50				
10	Cultivating	1	00				
25	Cultivating	1	00				
June 20	Hilling potatoes	2	50				
July 5	Spraying potatoes	1	85				
Sept. 29	Harvesting corn	7	25				
Oct. 5	Harvesting potatoes	3	75				
	Total	30	35				
	Profit	35	65				
	Check	66	00				

The profit is entered in the expenditures column. It is found by subtracting the expenditures from the receipts. When the profit and the expenditures are added, the result is the total receipts. This serves as a check on the accuracy of the computation.

EXERCISE 4

For each of the following examples that you do, prepare a sheet of paper by drawing the necessary **horizontal** and **vertical lines** as in the illustrative example above. The horizontal lines make **right angles** with the vertical lines. The horizontal lines are **parallel**; also the vertical lines are parallel.

1. Prepare an account for a boy's lemonade and ice cream stand for three days of a Chautauqua meeting.

August 11, cash on hand, \$5.75; bought 5 lb. sugar @ 11¢ per pound; 3 doz. lemons @ 48¢ per dozen; 2 gross lemonade straws @ 15¢ per gross; 3 hundred ice cream cones @ 75¢ per hundred. August 12, bought ice, 25¢; 3 gal. ice cream @ \$1.35 per gallon; sold 96 glasses lemonade @ 5¢ per glass, and 175 cones @ 5¢ each. August 13, bought ice, 25¢; 5 gal. ice cream @ \$1.35 per gallon; 2 doz. lemons @ 48¢ per dozen; 10 lb. sugar @ 11¢ per pound; 2 hundred cones @ 75¢ per hundred. Sold 127 glasses lemonade, and 218 cones. August 14, bought ice, 25¢; 2 gal. ice cream; 1 doz. lemons; 5 lb. sugar; 1 hundred cones. Sold 88 glasses lemonade, and 169 cones.

2. Prepare an account for a boy's newspaper business for one week. He buys papers for 1¢ each and sells them for 2¢ each. Monday, cash on hand, 83¢; bought 25 papers and sold 25. Tuesday, bought 35 papers and sold 33. Wednesday, bought 30 and sold 30. Thursday, bought 35 and sold 35. Friday, bought 40 and sold 36. Saturday, bought 40 and sold 38.

3. A family has the following record of receipts and expenditures on account of their strawberry field. Prepare a proper account and balance it on the date of the last item.

Apr. 25, removing straw and cleaning field, $3.50. May 2, bought 125 crates @ 5¢ each; also 2,000 boxes @ $4.00 per thousand. June 25, paid pickers $1.60, and sold 5 crates of berries @ $4.25 each. June 26, paid pickers $2.56, and sold 8 crates @ $4.25 each. June 27, paid pickers $3.20, and sold 10 crates @ $4.00 each. June 29, paid pickers $4.80, and sold 15 crates @ $3.25 each. June 30, paid pickers $5.12, and sold 16 crates @ $3.00 each. July 1, paid pickers $5.12, and sold 15 crates @ $2.85 each. July 2, paid pickers $3.84, and sold 12 crates @ $2.80 each. July 3, paid pickers $2.56, and sold 8 crates @ $2.90 each. July 5, paid pickers $1.92, and sold 6 crates @ $2.90 each. July 7, paid pickers $1.28, and sold 6 crates @ $3.15 each. July 9, sold 3 crates @ $3.25 each. July 11, sold 2 crates @ $3.25 each. July 14, sold 1 crate @ $3.50.

4. Prepare an account and find the balance from the following data: Sept. 1, 1920, balance from August, $28.75; salary received, $175.00; paid rent, $42.50. Sept. 2, paid grocer, $35.25; meat market man, $12.75; telephone company, $1.50. Sept. 10, paid gas and electric light company, $6.35. Sept. 12, bought suit clothes, $45.00. Sept. 15, received interest on Liberty Bonds, $21.30. Sept. 20, paid for coal, $30.00. Sept. 21, paid installment on Fourth Liberty Bond, $20.00. Sept. 22, received for extra work, $25.00. Sept. 29, paid maid, $30.00. Balance the account on Sept. 30.

5. From the following data, draw up an account for a girl working as a maid. Oct. 1, balance from September, $73.85. Oct. 4, received pay, $7.00. Oct. 9, spent for

moving picture show 17¢, and candy 15¢. Oct. 11, received $7.00. Oct. 14, bought shoes $6.50. Oct. 16, spent for carfare 12¢ and picture show 17¢. Oct. 18, received $7.00. Oct. 19, church contribution 25¢. Oct. 23, bought stockings, 60¢; house dress, $3.79; apron, $.89; ribbon, $.75; carfare, 12¢. Oct. 25, received $7.00. Oct. 26, church, 15¢. Oct. 30, bought winter suit, $42.50. Nov. 1, received $7.00. Balance the account on Nov. 2.

6. If you have a regular allowance or if you earn money while attending school, keep a record of your own receipts and expenditures for one month. Then draw up an account like that in Example 5.

EXERCISE 5

Computation Drill

1. How many of the following columns can you add in eight minutes?

a	*b*	*c*	*d*	*e*	*f*	*g*	*h*	*i*
784	280	637	743	469	418	694	418	637
963	504	794	247	634	532	387	870	216
842	926	580	539	719	279	256	674	206
769	325	306	672	582	531	318	506	472
374	496	368	138	462	648	579	568	947
428	837	862	637	416	245	275	620	372
657	627	648	492	586	837	546	764	563
948	518	374	597	397	168	679	425	679
375	721	223	264	578	257	375	364	472
284	293	863	146	295	739	462	875	412
625	148	430	587	486	616	496	462	607
397	372	573	823	312	583	563	507	562

2. Give orally or write only the remainders when 8 is subtracted from:

97 86 54 45 73 68 39 22

For further drill subtract also 7, 9, 6, and 5.

3. Subtract the lower number from the upper number:

a	*b*	*c*	*d*	*e*	*f*
63,894	30,586	47,296	285.679	47.298	64.08
52,625	24,869	35,072	205.084	28.096	38.268

4. Multiply each of the following numbers by 7:

84 63 97 45 56 72 38 29

5. Divide each of the following numbers by 6, giving the quotient and the remainder

26 34 45 73 58 97 62 89

6. Divide each of the numbers in Example 5 by 9.

7. Divide each of the numbers in Example 2 by 8.

8. Divide each of the numbers in Example 4 by 7.

9. Add the numbers in each part of Example 3.

4. **Making change.** — If a \$5.00 bill is offered in payment for purchases amounting to \$1.68, a clerk computes the change as follows:

\$1.68 and 2 *pennies* = \$1.70; and a *nickel* = \$1.75; and a *quarter* = \$2.00; and a \$2.00 bill = \$4.00; and a \$1.00 bill = \$5.00. The change then consists of 2 pennies, 1 nickel, 1 quarter, 1 \$2.00 bill and 1 \$1 bill, — a total of \$3.32.

EXERCISE 6

What coins and bills should you receive as change if your purchase of:

1. \$4.32 is paid for out of a \$5.00 bill.
2. \$3.87 is paid for out of a \$5.00 bill.
3. \$2.65 is paid for out of a \$10.00 bill.
4. \$.87 is paid for out of a \$2.00 bill.
5. \$3.24 is paid for out of a \$10.00 bill.

Imagine that you are a clerk in a store. Determine the change you should give a buyer who offers:

6. A \$5.00 bill after purchasing articles that cost 18¢, 33¢, and \$1.15.
7. A \$2.00 bill after purchasing articles that cost 12¢, 54¢, and 85¢.
8. A \$10.00 bill after purchasing articles that cost \$1.75, \$2.34, \$3.28, and 69¢.
9. A \$5.00 bill after purchasing articles that cost \$2.38, 17¢, 34¢, and 69¢.
10. A \$20.00 bill after purchasing articles that cost \$9.72, \$2.48, and \$1.75.

5. Invoices. — When a merchant sells goods, he usually gives the purchaser an itemized list of the articles bought, their cost, and the time and terms on which payment is expected. This statement is called an **invoice.** Bring to class any invoices or blanks on which they are prepared, that you can obtain in your town.

SAMPLE INVOICE

NORWICH, ILLINOIS
Feb. 15, 1922

Mr. *Arthur Anderson*
R. F. D. #2, Norwich, Illinois

Bought of Norwich Coal and Lumber Company

Terms. — Cash in 30 days.

2/15/22	*3 T. #2 lump coal*	*@ $5.75*		*17*	*25*
	2 rolls roofing paper	*@ 3.15*		*6*	*30*
	3 bundles shingles	*@ 1.75*		*5*	*25*
				28	*80*

REC'D PAYMENT
March 5, 1922
NORWICH COAL AND LUMBER CO.

EXERCISE 7

Draw the necessary lines to make a neat appearing invoice. Fill out an invoice from each of the following sets of data.

1. William Barton, 294 Elm Street, Lexington, Va., on September 15th, this year, bought of the Daniels Milling Company: 50 lb. wheat at 4¢ per pound; 50 lb. oats at $2\frac{1}{2}$¢ per pound; 30 lb. barley at $3\frac{1}{2}$¢ per pound, and 80 lb. cracked corn at $3\frac{1}{2}$¢ per pound. Receipt it as if it was paid on September 15th.

2. Fred Gilson of your city (supply an address), manager of a baseball team, ordered from the Fielding Sporting

Goods Co., Chicago, Ill., on June 18th, of this year, the following goods: ½ doz. Spalding balls @ $1.25 each; 2 bats @ 60¢ each; 1 bat @ 85¢; 2 bats @ 72¢ each; 9 complete uniforms @ $3.65 each; 1 catcher's mitt @ $4.25; 1 first baseman's mitt @ $3.50; 7 gloves @ $1.90 each; 1 mask @ $2.85; 1 body protector @ $3.75; 9 pairs shoes @ $2.50 each. Prepare the invoice and the receipt if the bill was paid on July 1.

3. Mr. Charles Nelson, milk dealer, sold to Mr. Robert Simpson 28 quarts of milk at 13¢ and 5½ quarts of cream at 50¢. Supply addresses and name of town. Date it October 1st. Receipt it as if it was paid on October 5th.

4. Edward Morris delivered to Mr. E. A. Smith during August of this year 27 daily papers at 2¢ each and 4 Sunday papers at 8¢ each. Date the statement Sept. 2, from your own town. Supply addresses.

5. Edward Arnold decides to refurnish his wife's kitchen. They purchase from the Model General Store the following goods: 1 kitchen range @ $59.65; 1 four-burner oil stove @ $28.90; 4 chairs @ $1.37 each; 1 table @ $10.25; 8 yds. of linoleum @ $3.60 per yard; 1 kitchen cabinet @ $31.45; 1 power washing machine @ $15.50. Make out the invoice for this equipment. Supply addresses for the purchaser and the store; also a date.

6. Monthly statements as issued by merchants are like invoices in some respects. Prepare a statement from the following data:

The Central Hardware Company, 28 Main Street, Newport, Illinois, sent this statement to Edward Ames, 463 So. Oak Street, on May 1, 1920. April 1, 1920, bill rendered (for March) $16.85. April 11, 10 lb. nails @ 6¢ per pound; 50 ft. wire @ 12¢ per foot. April 20, 1 pail @ $1.15; 1 dish pan @ $1.25. April 27, 2 lb. staples @ 15¢

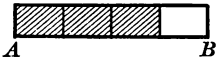
per pound; 1 roll chicken wire @ $4.85. Mr. Ames paid $15.00 on April 5th. After finding the total amount he owed the hardware company at the end of April, enter the amount which he paid on April 5th, and find the balance he owes at the end of April.

II. REVIEW OF COMMON FRACTIONS

6. The numerator and the denominator of a common fraction, together, are called the **terms** of the fraction.

$\frac{3}{4}$ ← *Numerator*
$\frac{3}{4}$ ← *Denominator*

A fraction like $\frac{3}{4}$ may be understood as meaning three of four equal parts into which a unit is divided. Thus the shaded part of rectangle AB represents $\frac{3}{4}$ of AB.



Mathematicians think of a fraction as the numerator divided by the denominator.

Thus, $\frac{3}{4}$ means $3 \div 4$.

A **proper fraction** is a common fraction whose numerator is less than its denominator. An **improper fraction** is a common fraction whose numerator is greater than its denominator.

A **mixed number** is an integer plus a fraction; as, $5+\frac{3}{4}$, usually written $5\frac{3}{4}$.

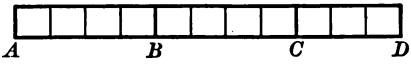
EXERCISE 8

1. Write three proper fractions.
2. Write three improper fractions.
3. Write three mixed numbers.
4. Illustrate the fraction $\frac{3}{5}$ by a drawing of a rectangle.
5. Illustrate by a drawing the fraction $\frac{1}{2}$; and, on the same drawing, the fraction $\frac{3}{6}$. How does the fraction $\frac{3}{6}$ compare with the fraction $\frac{1}{2}$?

6. Illustrate by a drawing the fraction $\frac{2}{3}$; and, on the same drawing, the fraction $\frac{4}{6}$. How does the fraction $\frac{4}{6}$ compare with the fraction $\frac{2}{3}$?

7. Improper fractions and mixed numbers.

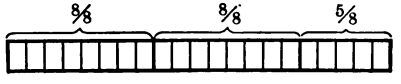
In the figure below, AB and BC each represent 1, and CD represents $\frac{3}{4}$ of 1. Then AD represents $2\frac{3}{4}$.



But since $1 = \frac{4}{4}$, then $2 = \frac{8}{4}$, and $2\frac{3}{4} = \frac{8}{4} + \frac{3}{4}$, or $\frac{11}{4}$.

This is called *changing a mixed number into an improper fraction.*

Again, $\frac{21}{8}$ means $21 \div 8$. Every eight $\frac{1}{8}$th's makes one whole. Dividing 21 by 8 gives 2 as quotient and 5 as remainder.



Hence $\frac{21}{8} = 2\frac{5}{8}$.

This is called *changing an improper fraction to a mixed number.*

EXERCISE 9

Express the following mixed numbers as improper fractions:

1. $3\frac{5}{8}$	4. $6\frac{7}{8}$	7. $18\frac{2}{3}$	10. $12\frac{1}{16}$	13. $83\frac{1}{3}$
2. $7\frac{2}{3}$	5. $11\frac{1}{4}$	8. $25\frac{1}{6}$	11. $22\frac{5}{12}$	14. $67\frac{3}{4}$
3. $9\frac{3}{5}$	6. $15\frac{3}{8}$	9. $32\frac{3}{7}$	12. $37\frac{1}{2}$	15. $21\frac{2}{3}$

Express the following improper fractions as mixed numbers. (See illustrative example § 7.)

16. $\frac{29}{4}$	19. $\frac{77}{8}$	22. $\frac{83}{5}$	25. $\frac{185}{12}$	28. $\frac{847}{64}$
17. $\frac{36}{7}$	20. $\frac{38}{8}$	23. $\frac{95}{4}$	26. $\frac{217}{16}$	29. $\frac{273}{25}$
18. $\frac{42}{5}$	21. $\frac{63}{10}$	24. $\frac{127}{8}$	27. $\frac{359}{32}$	30. $\frac{469}{50}$

8. Two Important Laws of Fractions.

In Example 6 of Exercise 8, it is observed that $\frac{2}{3}=\frac{4}{6}$. Notice that $\frac{4}{6}$ can be obtained from $\frac{2}{3}$ by multiplying both terms of $\frac{2}{3}$ by 2.

Similarly $\frac{1}{2}=\frac{3}{6}$. $\frac{3}{6}$ can be obtained from $\frac{1}{2}$ by multiplying both terms of $\frac{1}{2}$ by 3.

Law I. — The numerator and the denominator of a fraction may each be multiplied by the same number without changing the value of the fraction.

Law II. — The numerator and the denominator of a fraction may each be divided by the same number without changing the value of the fraction.

Thus $\frac{8}{24}=\frac{1}{3}$. (Divide both terms by 8.)

EXERCISE 10

1. Multiply both terms of the fraction $\frac{2}{3}$ by:

a. 2 *b.* 3 *c.* 5 *d.* 8 *e.* 10

How do all the resulting fractions compare?

2. By what must the terms of the fraction $\frac{3}{5}$ be multiplied to give the fraction $\frac{9}{15}$?

3. Divide both numerator and denominator of $\frac{20}{30}$ by:

a. 2 *b.* 5 *c.* 10

4. By what number can you divide both terms of the fraction $\frac{28}{70}$? What fraction do you obtain? How does it compare with $\frac{28}{70}$?

5. Multiply both terms of the fraction $\frac{5}{6}$ by some number. How does the resulting fraction compare with $\frac{5}{6}$? Why?

9. Reducing a fraction to its lowest terms.

Rule. — **To reduce a fraction to lower terms, divide its numerator and denominator by the same number.**

Thus $\frac{22}{44} = \frac{11}{44}$, dividing both terms by 2.

When the numerator and the denominator cannot both be divided exactly by any one number, the fraction is in its lowest terms.

Thus, $\frac{3}{4}$, $\frac{7}{8}$, $\frac{5}{8}$ are in their lowest terms.

$\frac{10}{15}$ is not in its lowest terms, for both 10 and 15 can be divided by 5, giving the fraction $\frac{2}{3}$.

EXERCISE 11

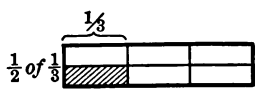
Reduce to lowest terms:

1. $\frac{20}{30}$	6. $\frac{76}{132}$	11. $\frac{15}{24}$	16. $\frac{10}{25}$	21. $\frac{9}{21}$
2. $\frac{15}{45}$	7. $\frac{30}{64}$	12. $\frac{18}{63}$	17. $\frac{12}{27}$	22. $\frac{10}{26}$
3. $\frac{12}{34}$	8. $\frac{18}{26}$	13. $\frac{49}{70}$	18. $\frac{15}{39}$	23. $\frac{21}{35}$
4. $\frac{14}{32}$	9. $\frac{6}{18}$	14. $\frac{28}{42}$	19. $\frac{14}{35}$	24. $\frac{35}{40}$
5. $\frac{10}{12}$	10. $\frac{12}{32}$	15. $\frac{56}{124}$	20. $\frac{18}{34}$	25. $\frac{6}{42}$

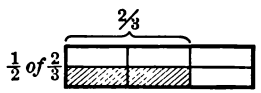
10. Multiplication of fractions.

Development. —

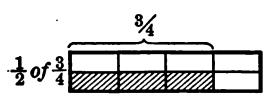
1.



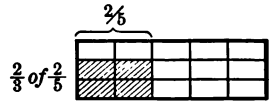
a. $\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$



b. $\frac{1}{2} \times \frac{2}{3} = \frac{2}{6}$



c. $\frac{1}{2} \times \frac{3}{4} = \frac{3}{8}$.



d. $\frac{2}{3} \times \frac{2}{5} = \frac{4}{15}$.

2. Each of these figures represents geometrically the product written immediately below the figure. All four multiplications suggest the following rules:

Rule 1. — **To multiply two or more fractions, multiply their numerators for the numerator of the product, and their denominators for the denominator of the product.**

2. If any numerator *and* denominator can each be divided by the same number, divide them by that number before multiplying. This is commonly called canceling.

Example 1. — $\frac{\cancel{3}^{1}}{\cancel{5}_{1}} \times \frac{\cancel{10}^{2}}{\cancel{9}_{3}} = \frac{2}{3}$

Here 3 and 9 are each divided by 3; also 5 and 10 are each divided by 5.

Example 2. — $3 \times \frac{4}{5} \times 2\frac{2}{3} = \frac{\cancel{3}^{1}}{\cancel{3}_{1}} \times \frac{4}{5} \times \frac{8}{\cancel{3}_{1}} = \frac{32}{5} = 6\frac{2}{5}$.

Here $2\frac{2}{3}$ is first changed to an improper fraction.

Example 3. — Multiply 42 by $12\frac{2}{3}$.

42
$12\frac{2}{3}$
$14 = \frac{1}{3}$ of 42
$14 = \frac{1}{3}$ of 42
84
42
532

First $\frac{1}{3}$ of 42 is found. Below that a second $\frac{1}{3}$ of 42 is written, since $\frac{2}{3}$ of 42 is wanted. Then 42 is multiplied by 12 and the results added.

NOTE. — This method is useful when the multiplicand or the multiplier is an integer. If both are mixed numbers, it is preferable for beginners to change each to an improper fraction.

EXERCISE 12

Represent by a geometrical figure, as on p. 18:

1. $\frac{1}{3}$ of $\frac{1}{2}$
2. $\frac{1}{4}$ of $\frac{1}{3}$
3. $\frac{1}{2}$ of $\frac{3}{5}$
4. $\frac{1}{3}$ of $\frac{2}{5}$
5. $\frac{2}{3}$ of $\frac{4}{5}$
6. $\frac{3}{4}$ of $\frac{3}{5}$

Find the following products, using the methods illustrated in Examples 1, 2, and 3, p. 19.

7. $\frac{3}{5}\times\frac{7}{8}$
8. $\frac{5}{4}\times\frac{7}{11}$
9. $\frac{2}{7}\times\frac{21}{8}$
10. $\frac{5}{8}\times\frac{4}{15}$
11. $\frac{7}{16}\times\frac{24}{35}$
12. $\frac{2}{3}\times\frac{21}{10}$
13. $\frac{11}{32}\times\frac{4}{3}$
14. $9\times\frac{2}{3}$
15. $4\times\frac{5}{8}$
16. $\frac{3}{4}$ of 12
17. $\frac{4}{5}$ of 15
18. $\frac{3}{8}$ of $\frac{2}{3}$
19. $\frac{5}{6}\times4\frac{1}{2}$
20. $3\frac{1}{3}\times\frac{1}{5}$
21. $10\frac{1}{4}\times8$
22. $12\frac{1}{2}\times\frac{2}{5}$
23. $16\frac{2}{3}\times9$
24. $37\frac{1}{2}\times4$
25. $33\frac{1}{3}\times9$
26. $6\frac{1}{4}\times\frac{8}{25}$
27. $5\times\frac{12}{15}\times\frac{3}{4}$
28. $\frac{7}{8}$ of $4\times1\frac{1}{5}$
29. $65\times12\frac{1}{5}$
30. $24\times16\frac{1}{2}$
31. $8\frac{1}{3}\times96$
32. $256\times3\frac{1}{8}$
33. $2\frac{1}{2}\times3\frac{1}{3}$
34. $4\frac{3}{8}\times2\frac{2}{3}$
35. $16\frac{2}{3}\times9\times3\frac{3}{4}$
36. $37\frac{1}{2}\times15\times8\frac{1}{4}$
37. $3\frac{1}{7}\times4\frac{1}{5}\times4\frac{1}{8}$
38. $625\times16\frac{2}{5}$
39. $450\times27\frac{2}{3}$

11. Problems using common fractions are very common in the daily work of clerks in stores and of workmen of all kinds.

In stores, clerks are commonly supplied with computing scales, but the purchaser should try to compute mentally the amount of the purchase. In the following problems, therefore, do as many as possible mentally; use paper and pencil only if necessary.

EXERCISE 13

Find the cost of:

1. 5 lb. of sugar @ $10\frac{1}{2}¢$ per pound.
2. $\frac{3}{4}$ lb. of cheese @ $35¢$ per pound.
3. $\frac{1}{2}$ lb. of lard @ $45¢$ per pound.
4. $1\frac{1}{2}$ lb. of steak @ $32¢$ per pound.
5. $2\frac{3}{4}$ lb. of chicken @ $38¢$ per pound.
6. $5\frac{1}{4}$ lb. roast of pork @ $30¢$ per pound.

7. 10 oz. (that is $\frac{5}{8}$ lb.) of bacon @ 45¢ per pound.
8. 12 oz. dried beef at 60¢ per pound.
9. 14 oz. boiled ham at 65¢ per pound.
10. $2\frac{1}{4}$ lb. of steak at 35¢ per pound.
11. $\frac{2}{3}$ yd. of embroidery at $3.75 per yard.
12. $\frac{4}{5}$ yd. of silk at $4.15 per yard.
13. $3\frac{1}{2}$ yd. dress goods at $5.25 per yard.
14. $11\frac{1}{4}$ yd. of silk braid at 85¢ per yard.
15. $2\frac{1}{2}$ yd. insertion at 37¢ per yard.
16. 6 in. ($\frac{1}{6}$ of a yd.) embroidery at $8.75 per yard.
17. $5\frac{1}{2}$ yd. dress goods at $3.75 per yard.
18. $\frac{1}{3}$ yd. silk at $6.25 per yard.
19. $3\frac{3}{4}$ lb. of sugar at 11¢ per pound.
20. $4\frac{3}{8}$ lb. of chicken at 28¢ per pound.
21. $15\frac{3}{5}$ tons coal at $7.50 per ton.
22. $21\frac{2}{3}$ acres of land at $175 per acre.
23. $92\frac{1}{2}$ feet iron pipe at $11\frac{1}{2}$¢ per foot.
24. 750 lb. of beef at $22\frac{1}{4}$¢ per pound.
25. 375 bu. of corn at 1.34\frac{1}{2}$ per bushel.
26. How much does a workman receive for working 8 hours if he is paid $37\frac{1}{2}$¢ per hour?
27. How much does one receive who is paid $62\frac{1}{2}$¢ per hour?
28. How much does one receive who is paid $87\frac{1}{2}$¢ per hour?
29. What is the cost of 9 gallons of gasoline at $24\frac{1}{2}$¢ per gallon?
30. What is the cost of 75 lb. of barley at $3\frac{1}{4}$¢ per pound?

EXERCISE 14

Problems Met in Adjusting Recipes

1. A recipe for "golden corn cake" calls for $\frac{3}{4}$ cup (c) of corn meal; $1\frac{1}{2}$ c of sugar; 4 teaspoonfuls (t) of baking powder; $\frac{1}{2}$ t of salt; 1 c of milk; 1 egg; and 1 tablespoonful (T) of melted butter.

a. Find the quantity of each ingredient required for one half of the recipe. (Multiply each quantity by $\frac{1}{2}$.)

b. Find the quantities required for one third of the recipe.

2. The following recipe is given for coffee sauce to be served with ice cream:

$1\frac{1}{2}$ c of milk; $\frac{1}{2}$ c of ground coffee; $\frac{1}{3}$ c of sugar; $\frac{3}{4}$ T of arrowroot; and salt.

How much of each ingredient should be used to make one half the recipe? Two thirds the recipe?

3. A recipe for "orange snow pudding" is 1 T of gelatine; $\frac{1}{4}$ c of cold water; $\frac{3}{4}$ c of boiling water; $\frac{1}{2}$ c of sugar; $\frac{3}{4}$ c of orange juice; 2 T of lemon juice; and the white of 1 egg. What quantities should be used to make $\frac{1}{3}$ portion of this recipe? $1\frac{1}{2}$ portions?

4. A recipe for plum preserves calls for three fourths as much sugar as fruit, and one cup of water to each pound of sugar. How much sugar and how much water are required for:

a. 12 lb. of fruit? *b.* 9 lb.? *c.* 14 lb.?

5. A recipe for waffles is:

$1\frac{3}{4}$ c flour; 3 t baking powder; 2 eggs; 1 T melted butter; $\frac{1}{2}$ t salt; 1 c milk.

a. What quantities should be taken to make one half portion of this recipe? *b.* $2\frac{1}{3}$ portions?

12. Division of fractions.

Development. — 1. If the fraction $\frac{2}{3}$ is turned over, the result is the fraction $\frac{3}{2}$. The fraction $\frac{2}{3}$ is said to be **inverted.**

Thus, $\frac{3}{5}$ inverted gives $\frac{5}{3}$.
$\frac{1}{4}$ inverted gives $\frac{4}{1}$ or 4.
3 inverted gives $\frac{1}{3}$, for 3 is the same as $\frac{3}{1}$

2. Consider the following quotients:

$$a.\ 2\overline{)24}=12 \qquad b.\ 4\overline{)24}=6 \qquad c.\ 6\overline{)24}=4$$

In *b* the divisor is *twice* as great as in *a* and the quotient is *one half* as great as in *a*.

In *c* the divisor is *three* times as great as in *a* and the quotient is *one third* as great as in *a*.

Similarly, if a divisor be multiplied by 4, the quotient will be divided by 4.

3. $1\div\frac{1}{3}=3$. So $1\div\frac{2}{3}=\frac{1}{2}$ of 3 or $\frac{3}{2}$.

4. Since $1\div\frac{2}{3}=\frac{3}{2}$, then $5\div\frac{2}{3}=5\times\frac{3}{2}$ or $\frac{15}{2}$
and $7\div\frac{2}{3}=7\times\frac{3}{2}$ or $\frac{21}{2}$.

These last examples suggest the

Rule. — To divide a number by a fraction, invert the divisor and multiply.

Remember that the divisor is the number *following* the division sign.

$$\text{Thus } \frac{7}{16}\div\frac{3}{4}=\frac{7}{\cancel{16}_{4}}\times\frac{\cancel{4}^{1}}{3}=\frac{7}{12}.$$

$$\frac{5}{8}\div 3=\frac{5}{8}\times\frac{1}{3}=\frac{5}{24}.$$

EXERCISE 15

Find the quotients.

1. $\frac{5}{8} \div \frac{3}{16}$
2. $\frac{4}{5} \div \frac{3}{10}$
3. $\frac{5}{6} \div \frac{2}{9}$
4. $\frac{7}{8} \div \frac{9}{24}$
5. $\frac{3}{4} \div \frac{9}{8}$
6. $\frac{3}{8} \div \frac{6}{20}$
7. $4 \div \frac{2}{3}$
8. $7 \div \frac{5}{8}$
9. $6 \div \frac{3}{5}$
10. $\frac{5}{3} \div \frac{2}{1}$
11. $\frac{3}{8} \div 4$
12. $\frac{5}{6} \div 3$
13. $16 \div \frac{3}{4}$
14. $\frac{13}{16} \div \frac{1}{2}$
15. $\frac{9}{32} \div \frac{3}{2}$
16. $24 \div \frac{3}{8}$
17. $\frac{12}{25} \div \frac{4}{15}$
18. $\frac{5}{8} \div \frac{3}{2}$
19. $\frac{7}{12} \div \frac{1}{4}$
20. $\frac{15}{24} \div \frac{10}{18}$
21. $\frac{16}{35} \div \frac{1}{5}$
22. $\frac{4}{5} \div 2$
23. $\frac{7}{9} \div 3$
24. $\frac{5}{9} \div \frac{2}{15}$
25. $3\frac{3}{5} \div \frac{8}{7}$
26. $12 \div 1\frac{1}{2}$
27. $13\frac{1}{3} \div 4\frac{1}{6}$
28. $27 \div 3\frac{3}{4}$
29. $16\frac{1}{2} \div 3\frac{2}{3}$
30. $18\frac{3}{4} \div 3\frac{4}{7}$
31. $66\frac{2}{3} \div 11\frac{1}{9}$
32. $42\frac{6}{7} \div 2\frac{1}{7}$
33. $125 \div 15\frac{5}{8}$
34. $44\frac{4}{9} \div 6\frac{2}{3}$

35. *a.* How many curtains can be cut from 50 yards of curtain material if each curtain requires 2 yards?

b. If you know how much material is required to make one article of a certain kind, and have a quantity of that material, how can you determine the number of articles of that kind that can be made from the material on hand?

36. How many towels $\frac{5}{6}$ yd. long can be cut from 10 yards of the material?

37. How many garments requiring $2\frac{1}{2}$ yd. (that is $\frac{5}{2}$ yd.) of material can be cut from 15 yards of the material?

38. How many aprons $\frac{3}{4}$ yd. in length can be cut from 12 yd. of the material?

39. It takes $6\frac{1}{4}$ yd. of lace to trim a certain garment. How many garments can be trimmed from a 25 yd. bolt of lace?

HINT. — $6\frac{1}{4}$ is how many 4ths?

40. How many pieces of wood $2\frac{1}{2}$ ft. long can be cut from a 16 foot board?

41. How many floor boards $5\frac{1}{4}$ inches wide does it take to cover a room 13 feet wide?

HINTS. — 13 ft. = how many inches? $5\frac{1}{4}$ = how many 4ths?

42. How many floor boards $3\frac{1}{4}$ inches wide does it take to cover a room 14 ft. 6 in. wide?

43. How many pieces $2\frac{1}{4}$ ft. long can be cut from a board 14 ft. in length?

44. If a family uses $\frac{3}{5}$ ton of coal in a week, how long will 12 tons last them at that rate?

45. If a family uses $\frac{4}{7}$ gal. of gasoline per day for cooking and lighting gas, how long will a barrel containing 54 gallons last?

46. A boy feeds $5\frac{1}{2}$ pounds of grain each day to his chickens. How long will it take him to use all of 100 pounds of chicken feed? (Give only the number of whole days for which he will have enough feed.)

47. A man who had used all the gasoline in his car stopped to buy more. He found that the gasoline would cost him $29\frac{1}{2}¢$ per gallon. He had only \$1.15 in cash. How many full gallons could he buy?

48. A man decides to spend \$150 for coal. Coal is selling for $\$7\frac{1}{2}$ per ton. How many tons may he order?

13. Expressing fractions as equal fractions with larger denominators.

Rule. — To express a fraction as an equal fraction with a larger denominator, multiply the numerator and the denominator of the given fraction by the same number. This number is found by dividing the new denominator by the old denominator.

Example. — Express $\frac{2}{3}$ with the denominator 9.

$9 \div 3 = 3$. Now multiply both terms of $\frac{2}{3}$ by 3. Then $\frac{2}{3} = \frac{6}{9}$.
Similarly $\frac{5}{6} = \frac{10}{12}$, multiplying both terms by 6.
$\frac{7}{8} = \frac{14}{16}$, multiplying both terms by 2.

EXERCISE 16

1. Express $\frac{2}{3}$
a. as 12ths; *b.* as 18ths; *c.* as 9ths; *d.* as 24ths.

2. Express $\frac{3}{5}$
a. as 25ths; *b.* as 35ths; *c.* as 100ths; *d.* as 55ths.

3. Express $\frac{5}{6}$ as an equal fraction with denominator:
a. 12 *b.* 36 *c.* 24 *d.* 42 *e.* 96

4. Since $1 = 12$ twelfths, then $4 = 4 \times 12$ twelfths, or 48 twelfths.
That is, $4 = \frac{48}{12}$.
Similarly, express 4
a. as 8ths; *b.* as 6ths; *c.* as 10ths.

5. Express 7 as 21sts; as 14ths; as 28ths.

6. Express 8 as 9ths; as 4ths; as 8ths; as 16ths.

7. Express 5 as 3ds; as 10ths; as 7ths.

8. Express as 9ths: *a.* 4; *b.* $\frac{2}{3}$; *c.* $\frac{5}{3}$.

9. Express as 12ths: *a.* $\frac{1}{2}$; *b.* $\frac{3}{4}$; *c.* $\frac{2}{3}$; *d.* 5.

10. Express as 16ths: *a.* $\frac{5}{8}$; *b.* $\frac{3}{4}$; *c.* $\frac{1}{2}$; *d.* 2.

11. $\frac{3}{8}$ is how many 32ds? 12. $\frac{2}{3}$ is how many 18ths?

13. $\frac{2}{5}$ is how many 25ths? 14. $\frac{5}{6}$ is how many 30ths?

14. Addition and subtraction of fractions.
Before fractions can be added or subtracted, they must have the same denominator; this is called their **common denominator.**

Rule. — To add or subtract fractions:

1. Change the fractions into equal fractions having a common denominator.

2. Then add, or subtract, the numerators for the numerator of the result, and use the common denominator for the denominator of the result.

Example 1. — Add $\frac{2}{3}$ and $\frac{3}{4}$.

Solution. — 1. The *least* common denominator is 12.

$\frac{2}{3} = \frac{8}{12}$, and $\frac{3}{4} = \frac{9}{12}$. See § 13.

2. $\therefore \frac{2}{3} + \frac{3}{4} = \frac{8}{12} + \frac{9}{12} = \frac{17}{12} = 1\frac{5}{12}$.

Example 2. — Find $\frac{7}{8} - \frac{1}{3}$.

Solution. — 1. The least number that will contain both 8 and 3 is 24.

2. $\frac{7}{8} = \frac{21}{24}$ and $\frac{1}{3} = \frac{8}{24}$.

3. $\therefore \frac{7}{8} - \frac{1}{3} = \frac{21}{24} - \frac{8}{24} = \frac{13}{24}$.

EXERCISE 17

Find:

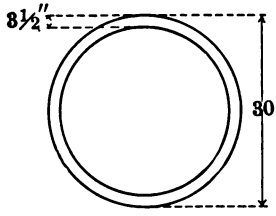
1. $\frac{1}{2}+\frac{1}{3}$	5. $\frac{2}{3}+\frac{1}{4}$	9. $\frac{3}{4}+\frac{5}{8}$	13. $\frac{5}{8}+\frac{5}{6}$
2. $\frac{1}{2}+\frac{1}{4}$	6. $\frac{3}{4}+\frac{1}{3}$	10. $\frac{4}{5}+\frac{2}{3}$	14. $\frac{2}{3}+\frac{3}{8}$
3. $\frac{1}{3}+\frac{1}{4}$	7. $\frac{2}{3}+\frac{1}{5}$	11. $\frac{5}{6}+\frac{7}{8}$	15. $\frac{3}{4}+\frac{4}{5}$
4. $\frac{2}{3}+\frac{1}{2}$	8. $\frac{3}{5}+\frac{1}{2}$	12. $\frac{3}{16}+\frac{3}{4}$	16. $\frac{5}{6}+\frac{2}{3}$
17. $\frac{2}{3}-\frac{1}{3}$	20. $\frac{9}{10}-\frac{3}{5}$	23. $2\frac{1}{2}-\frac{3}{4}$	26. $8-\frac{5}{9}$
18. $\frac{7}{8}-\frac{2}{5}$	21. $\frac{7}{8}-\frac{1}{3}$	24. $5\frac{3}{5}-\frac{7}{10}$	27. $\frac{5}{12}-\frac{3}{8}$
19. $\frac{5}{6}-\frac{3}{4}$	22. $5-\frac{3}{8}$	25. $6\frac{5}{8}-\frac{3}{4}$	28. $\frac{25}{32}-\frac{3}{4}$
29. $2\frac{1}{2}+3\frac{1}{4}$	31. $7\frac{5}{8}+6\frac{1}{4}$	33. $4\frac{3}{7}+2\frac{1}{2}$	35. $10\frac{3}{4}+6\frac{5}{8}$
30. $5\frac{2}{3}+2\frac{1}{4}$	32. $9\frac{3}{5}+2\frac{1}{2}$	34. $6\frac{3}{8}+5\frac{3}{5}$	36. $9\frac{4}{7}+3\frac{1}{3}$
37. $5\frac{3}{5}-2\frac{1}{10}$	39. $16\frac{2}{3}-11\frac{1}{4}$	41. $32\frac{5}{6}-15\frac{1}{4}$	43. $22\frac{3}{5}-16\frac{2}{3}$
38. $12\frac{1}{2}-8\frac{1}{3}$	40. $28\frac{3}{8}-15\frac{2}{3}$	42. $47\frac{3}{4}-18\frac{3}{4}$	44. $33\frac{1}{3}-19\frac{1}{4}$

45. A boy worked $1\frac{1}{3}$ hours on Monday, $1\frac{1}{4}$ hours on Tuesday, $1\frac{1}{2}$ hours on Wednesday, and $1\frac{1}{4}$ hours on Friday. How many hours did he work during the week? If he was paid 20¢ per hour, how much should he have received?

46. Find the cost of three loads of coal weighing $2\frac{1}{4}$, $2\frac{1}{8}$, and $2\frac{1}{2}$ tons respectively at $7.50 per ton.

47. On an electric light wire pole, the electricity wires are 15 feet above ground. $3\frac{1}{3}$ feet below them is a telephone wire. How high above ground is the telephone wire?

48. On an automobile wheel is a 30 inch by $3\frac{1}{2}$-inch tire, fully inflated. 30 inches is the distance from the point where the tire touches the ground to the top of the tire. $3\frac{1}{2}$ inches is the diameter of the inflated tire. What is the diameter of the rim of the wheel?



49. Why will a 31-inch by 4-inch tire fit the rim described in Example 48?

50. A man bought some shares in the ownership of a business, paying $\$76\frac{3}{8}$ for each share. He was compelled to sell them later for $\$74\frac{1}{4}$ for each share. How much did he lose on each share?

EXERCISE 18

Meat Market and Grocery Store Problems

1. What is the cost of a chicken weighing $5\frac{1}{4}$ pounds at 28¢ per pound?

2. What is the cost of $7\frac{3}{8}$ pounds of pork at 23¢ per pound?

3. A grocer agreed to pay $2\frac{3}{4}¢$ per pound for squash. How much should a gardener receive who sold him 328 pounds of squash?

4. Find the total cost of $1\frac{1}{2}$ lb. bacon @ $55¢$ per lb., $1\frac{3}{4}$ lb. sirloin steak at $36¢$ per lb., and $2\frac{1}{2}$ lb. lard at $38¢$ per lb. How much change should the purchaser receive from a $5.00 bill?

5. Find the total cost of a chicken weighing 4 lb. 10 oz. ($4\frac{5}{8}$ lb.) at $32¢$ per lb., and a slice of ham weighing 1 lb. 5 oz. at $40¢$ per lb.

6. Find the cost of $6\frac{3}{4}$ lb. of rib roast at $28¢$ per lb., and 1 lb. 6 oz. of sausage at $33¢$ per lb.

7. Prepare a sales statement for the following goods sold *to-day* by the Central Market to Mrs. E. M. Golden:

15 lb. potatoes at $3\frac{1}{2}¢$ per lb.; 5 lb. rice at $12\frac{1}{2}¢$ per lb.; 10 lb. corn meal at $5\frac{1}{4}¢$ per lb.; $3\frac{3}{4}$ lb. of cabbage at $5¢$ per lb.; 2 lb. coffee at $55¢$ per lb.; 10 bars soap at $6\frac{3}{4}¢$ per bar; $\frac{1}{2}$ lb. dried beef at $65¢$ per lb.; $\frac{3}{4}$ lb. cheese at $38¢$ per lb.

8. A market man bought of a farmer 5 chickens weighing $5\frac{1}{4}$, $4\frac{3}{8}$, $5\frac{3}{8}$, $4\frac{1}{2}$, and $5\frac{5}{16}$ pounds respectively. He agreed to pay $27¢$ per pound. (*a*) What did he owe the farmer? (*b*) The farmer bought a slice of round steak weighing 2 lb. 9 oz., at $24¢$ per lb. How much cash should he have received from the market man?

9. A woman bought for Thanksgiving Day dinner 1 fifteen-pound turkey at $37\frac{1}{2}¢$ per pound; 1 quart cranberries at $15¢$; 1 head of lettuce at $18¢$; 1 seven-pound pumpkin at $3\frac{1}{2}¢$ per pound; 15 pounds of potatoes at $2\frac{1}{3}¢$ per pound. What was the amount of her bill?

10. A class in a school bought the following material to make candy for a candy sale: 85 pounds of granulated sugar at $18\frac{1}{2}¢$ per pound; 4 pounds of chocolate at $44\frac{1}{4}¢$ per pound. What was the cost of these articles?

EXERCISE 19

Dry Goods Store Problems

1. Find the total cost of $6\frac{1}{2}$ yd. pillow tubing @ 45¢ per yd., and $8\frac{1}{3}$ yd. of sheeting at 86¢ per yd.

2. Find the total cost of $2\frac{2}{3}$ yd. of comforter silkaline at \$2.87 per yd.; $2\frac{2}{3}$ yd. of comforter cretonne at \$2.64 per yd.; and two 3-pound rolls cotton batting at \$1.49 per roll.

3. Find the cost of $4\frac{1}{4}$ yd. of foulard at 48¢ per yd., and $3\frac{3}{4}$ yd. percale at 53¢ per yd.

4. Find the cost of $6\frac{2}{3}$ yd. sateen at 69¢ per yd.

5. Find the cost of $1\frac{4}{5}$ yd. of white oil cloth at 78¢ per yd.

6. Find the cost of $5\frac{1}{2}$ yd. white organdie at 83¢ per yd., and $5\frac{1}{4}$ yd. Persian lawn at 62¢ per yd.

7. Estimate of material for a woman's coat: $3\frac{1}{2}$ yd. velour coating at \$4.25 per yd.; $3\frac{3}{4}$ yd. lining satin at \$1.55 per yd.; $1\frac{1}{2}$ yd. lining canvas at 55¢ per yd.; $\frac{3}{4}$ doz. large fancy buttons at \$2.25 per doz.; findings 50¢.

8. Estimate of cost of material for a wool dress: $2\frac{7}{8}$ yd. all wool serge at \$3.95 per yd.; $\frac{3}{4}$ yd. net for lining at 65¢ per yd.; $\frac{3}{4}$ yd. belting at 35¢ per yd.; $\frac{7}{8}$ yd. vesting brocade at \$3.75 per yd.; findings, 50¢.

9. Estimate of cost of material for a summer dress. $3\frac{3}{4}$ yd. white organdie at 95¢ per yard; 15 yd. valenciennes lace at 18¢ per yard; $1\frac{5}{8}$ yd. ribbon for girdle at 79¢; findings 50¢.

10. Bed sheets 90 inches wide and 95 inches long can be purchased for \$2.55. Sheeting 90 inches wide can be bought for 86¢ per yard. Compare the cost of one dozen ready-made sheets with the cost of enough sheeting to make one dozen sheets 95 inches long that have a 1-inch hem at the bottom and a $2\frac{1}{2}$-inch hem at the top.

III. MULTIPLICATION AND DIVISION OF DECIMALS

15. Reading and understanding large decimal numbers. — A number like 3.456 is read *three and four hundred fifty-six thousandths.*

.4 stands for $\frac{4}{10}$ or $\frac{400}{1000}$; .05 stands for $\frac{5}{100}$ or $\frac{50}{1000}$; and .006 stands for $\frac{6}{1000}$.

Then .456 represents $\frac{400}{1000}+\frac{50}{1000}+\frac{6}{1000}$ or $\frac{456}{1000}$.

A figure written in the

a. *first decimal place* represents *tenths ;*

b. *second decimal place* represents *hundredths ;*

c. *third decimal place* represents *thousandths ;*

d. *fourth decimal place* represents *ten thousandths ;*

e. *fifth decimal place* represents *hundred thousandths ;*

f. *sixth decimal place* represents *millionths.*

Then come ten millionths, hundred millionths, billionths, etc.

EXERCISE 20

Read the following:

1. .287	3. 2.4562	5. 4.279035	7. 78.097625
2. .9854	4. 3.95724	6. 52.00625	8. 10.00407

9. *a.* Read and compare the decimal fractions .2, .20, .200, etc.

b. Read and compare the decimal fractions .35, .350, .3500, etc.

c. What effect upon the value of a decimal fraction does it have to annex zeros to the end of it?

10. *a.* Read and compare .2, and .02. (2 is called the significant figure.)

b. Read and compare the value of .3 and .003.

c. Read and compare the value of .5 and .0005.

d. What is the effect of placing between the decimal point and the first significant figure: one zero? two zeros? three zeros?

Write the following numbers:

11. Eighty-two and twenty-five hundredths.

12. Seven and sixteen thousandths.

13. Seventy-six and twenty-three thousandths.

14. Eighty-four and one hundred twenty-nine ten thousandths.

15. One hundred ninety-two and three hundred forty-seven ten thousandths.

16. Two hundred thirty-eight hundred thousandths.

17. Eleven and six thousandths.

18. Five and fourteen ten thousandths.

19. Twenty-two and two hundred twenty-two thousandths.

20. Fifteen and six hundred eight millionths.

16. Multiplication of decimals.

Notice that $.3 \times .2 = \frac{3}{10} \times \frac{2}{10} = \frac{6}{100} = .06.$

$.4 \times .12 = \frac{4}{10} \times \frac{12}{100} = \frac{48}{1000} = .048.$

$.11 \times .15 = \frac{11}{100} \times \frac{15}{100} = \frac{165}{10000} = .0165$

These examples suggest the

Rule. — The number of decimal places in a product equals *the number in the* multiplier plus the number in the multiplicand.

It may be necessary to prefix zeros in the product as in the

Example 1. — Find $.764 \times .0108$.

In this example, there must be seven decimal places in the product. After multiplying, there are only the five figures 82512. Therefore two zeros are prefixed to these figures, giving the result .0082512.

$$\begin{array}{r} .764 \\ .0108 \\ \hline 6112 \\ 7640 \\ \hline .0082512 \end{array}$$

EXERCISE 21

Find the following products:

	a	*b*	*c*	*d*	*e*	*f*	*g*
1.	5.7	.68	23.4	32.6	46.05	24.8	84.93
	.7	.5	.4	.8	.9	.6	.9

	a	*b*	*c*	*d*	*e*	*f*	*g*
2.	3.61	5.84	5.7	.462	5.86	2.953	6.025
	.23	.18	2.9	.33	.027	3.26	4.07

	a	*b*	*c*	*d*	*e*	*f*	*g*
3.	5.246	3.05	.274	9.81	6.254	.074	8.231
	4.9	.024	5.61	2.4	4.83	6.29	5.04

4. *a.* Multiply each of the following numbers by 10:

7.8 2.045 925.43 67.257 8.296

b. How far and in what direction is the decimal point moved when the multiplicand is multiplied by 10?

5. *a.* Multiply each of the following numbers by 100:

3.52 1.86 45.237 68.064 74.8095

b. How far and in what direction is the decimal point moved when the multiplicand is multiplied by 100?

6. *a.* Multiply each of the following numbers by 1000:

3.257 53.279 47.081 25.86 94.3

b. How far and in what direction is the decimal point moved when the multiplicand is multiplied by 1000?

7. How many feet are there in 13 rods? (1 rd. = 16.5 ft.) In 80 rods?

8. How many square yards are there in 15.5 square rods? (1 sq. rd. = 30.25 sq. yd.)

9. If an electric light line can be built for 36.2¢ per foot, how much will a line 785 feet long cost?

10. What will a line 1135 feet long cost at 44.5¢ per foot?

11. What is the cost of 13 gallons of gasoline at 24.2¢ per gallon?

12. How wide a space will 43 boards cover, if each board is 5.25 inches wide?

Find the cost of the following wholesale purchases:

13. 3000 dozen eggs at $.533 per dozen.

14. 18,253 bushels of oats at $.7825 per bushel.

15. 785 tons of coal at $5.325 per ton.

16. 1254 tons of anthracite coal at $10.85 per ton.

17. 645.5 yards of wool serge dress goods at $3.2325 per yard.

18. 185 barrels of salt pork at $18.317 per barrel.

19. 1350 pounds of worsted yard at $.7875 per pound.

20. Give the rule for finding the cost of any number of articles of one kind when the cost of one article is known.

21. A cubic foot of water weighs approximately 62.5 pounds.

a. Gold weighs 19.26 times as much as water. What is the weight of a cubic foot of gold?

b. Ice weighs .92 times as much as water. What is the weight of a cubic foot of ice?

c. Air weighs .001293 times as much as water. What is the weight of a cubic foot of air?

22. The distance from one corner of a square field to the opposite corner is 1.414 times the length of one of the sides of the field. If the side of the field is 348 ft., how much shorter is the distance in a straight line from one corner to the opposite one than the distance between these two points along two sides of the field?

23. Many countries do not use our system of measures; instead, they use the metric system of measures. The unit of length, like our yard, is one meter. One meter equals 39.37 inches.

a. How many feet and inches are there in 10 meters?

b. For longer distances, the kilometer is used. A kilometer equals 1000 meters. How many feet are there in a kilometer? How many feet more is this than $\frac{3}{5}$ of a mile? (One mile has 5280 ft.)

24. In order to find the distance around a circular object, multiply the diameter of the object (the distance through or across it) by 3.1416.

a. What is the distance around the outside of a 30-inch automobile tire?

b. How far does the car go, while the rear wheel revolves 100 times?

Division of Decimals

17. Dividing a decimal number by an integer.

Notice that $.8 \div 2 = \frac{8}{10} \div 2 = \frac{4}{10} = .4.$

$$.045 \div 3 = \frac{45}{1000} \div 3 = \frac{15}{1000} = .015.$$

These, and other examples like them, suggest the

Rule. — When a decimal number is divided by an integer, the quotient has as many decimal places as the dividend.

EXERCISE 22

Divide:

1. .48 by 4	5. .069 by 3	9. 8.724 by 6
2. 2.79 by 3	6. .595 by 7	10. 89.739 by 13
3. 45.75 by 5	7. 9.472 by 8	11. 4.658 by 17
4. 6.284 by 2	8. 10.251 by 9	12. 92.529 by 23

18. Moving the decimal points in the dividend and the divisor.

Notice that the same quotient is obtained in each of the following divisions.

4.	4.	4.
a. 2.)8.	*b*. 20.)80.	*c*. 200.)800.

These results suggest the

Rule. — The dividend and the divisor may each be multiplied by 10, by 100, by 1000, etc., without changing the value of the quotient.

Consequently the decimal point of the divisor and the dividend may each be moved one place, or two places, or three places to the right without changing the value of the quotient. Similarly they can be moved to the left.

19. Dividing a number by a decimal divisor.

Example 1. — Divide 37.68 by .6

Solution. — Move the decimal point of the divisor one place to the right, making the divisor an integer. Place a cross over the original decimal point to show that it is crossed out. Do the same in the dividend.

Divide 37 by 6, placing the quotient 6 over the digit 7 of 37.

Divide 16 by 6, placing the quotient 2 over the digit 6 of 16.

Place the decimal point of the quotient over that of the dividend.

Divide 48 by 6, placing the quotient 8 over the digit 8 of 48.

```
      6 2.8
   ×6.)37×6.8
      36
      16
      12
       48
       48
```

The number of decimal places in the quotient must be the same as the number following the *new* decimal point of the dividend. In this example, that number is one.

Rule. — To divide a number by a decimal number:

1. Move the decimal point of the divisor to the right of the divisor, thus changing the divisor into an integer. Cross out the original decimal point.

2. Move the decimal point of the dividend an equal number of places to the right, annexing zeros if necessary. Cross out the original decimal point of the dividend.

3. Divide, placing each digit of the quotient directly above the last digit of the partial dividend that gives that digit of the quotient; place the decimal point of the quotient over that of the dividend.

4. The number of decimal places in the quotient equals the number in the dividend following its new decimal point.

Example 2. — Divide .06084 by 2.34

Solution. — Both decimal points are moved two places to the right. 234 is contained in 60 zero times.

Place 0 in the quotient over the digit 0 of 60. Similarly place 2 over 8 of 608, and 6 over the second 4 of 1404. The result is .026.

```
        .026
2×34.)×06.084
      4 68
      1 404
      1 404
```

EXERCISE 23

1. Divide each of the following numbers by .8:

a. 55.2 *b.* 6.48 *c.* .592 *d.* 8.72 *e.* 27.36

2. Divide each of the following numbers by .34:

a. 8.16 *b.* 304.98 *c.* .986 *d.* .04692 *e.* 56.168

Divide:

3. 7.75 by 2.5
4. 248.16 by 4.7
5. 1.5964 by .52
6. 16 by .25
7. 18.7731 by .027
8. 25.36482 by 4.03
9. 2183.6 by .824
10. 22.1325 by 6.81
11. 1.12848 by .048
12. 107.316 by 29.81
13. 69.212 by 5.72
14. 376.74 by 4.83
15. 162.1426 by 35.02
16. 260.186 by 31.73
17. 577.2305 by 2.35
18. 1452.9834 by 36.09
19. 277.7793 by 26.43
20. 7452.4 by 24.8

21. *a.* Divide each of the following numbers by 10:

92.5 8.67 54.3 654.2 .72 .065

b. Give a rule for dividing any number by 10.

22. *a.* Divide each of the following numbers by 100:

43.57 36.25 84.7 295.1 6.43 .86

b. Give a rule for dividing any number by 100.

c. Give a rule for dividing any number by 1000.

23. Divide 328.8 by 2400. (24.00)×(3.28)×8

(Move the decimal points two places to the left.)

Similarly divide:

24. 6945 by 300
25. 7896.3 by 3000
26. 578.42 by 200
27. 6972 by 4000
28. 258.55 by 500
29. 6934.26 by 600

30. 479.18 by 260 **32.** 764.4 by 420 **34.** 2357.8 by 2000
31. 3955 by 3500 **33.** 5493.76 by 640 **35.** 9643.75 by 2500

20. Dividing when the quotient is not exact.

Example 1. — Divide 28.93 by 4.58, carrying out the division to two decimal places.

Solution. — After the division is carried out to two places, there is still the remainder 302.

```
          6.31
    4.58)28.93.00
         27 48
         ------
          1 45 0
          1 37 4
          ------
            7 60
            4 58
            ----
            3 02
```

The result may be written 6.31^{+} or 6.31. The small plus sign has little value. If greater accuracy is desired than is given by two decimal places in the quotient, divide again and thus obtain the third decimal place in the quotient. In this example, the next figure would be 6, making the result 6.316.

EXERCISE 24

Find the following quotients to two decimal places:

1. $5430 \div 26$ **6.** $691.8 \div 71.74$ **11.** $23{,}456.84 \div 124.35$
2. $4639 \div 15$ **7.** $768.25 \div 2.7$ **12.** $16{,}008.7 \div 362.5$
3. $284.29 \div 38$ **8.** $437 \div 6.2$ **13.** $67{,}834.09 \div 172.21$
4. $19.763 \div 8.41$ **9.** $948.48 \div 25$ **14.** $2.0043 \div .874$
5. $462.9 \div 91.07$ **10.** $2346.50 \div 45.2$ **15.** $36.7281 \div 3.157$

NOTE. — How far to carry out any particular division, depends upon the problem. For example, if the quotient is an amount of money, three decimal places is all that will have any meaning in most cases.

EXERCISE 25

Finding Averages

1. From a field of 23 acres, 634 bushels of corn were harvested. What was the average amount harvested per acre, obtaining the result to one decimal place?

2. The sum of the weights of the eleven men on a football team was 1807.5 pounds. What was the average weight per man, to one decimal place?

3. A man used 23 tons of coal to heat his house during nine months of a recent year. What was the average number of tons used per month, to one decimal place?

4. A family found that its expense for food during a certain thirty-one day month had been \$78.45. What was the average expense per day, in dollars and cents?

5. The monthly bills for electricity for a family were \$4.45, \$4.54, \$4.36, \$4.12, \$3.95, \$3.65, \$3.49, \$3.45, \$3.60, \$3.88, \$4.10, and \$4.20. What was the average monthly expense for electricity?

6. The sales of a grocer during six days of one week were \$249.80, \$265.34, \$237.62, \$276.95, \$268.19, and \$315.48. What was the average of his sales per day?

7. The total monthly expenditures of a family during one year were \$234.75, \$215.80, \$195.46, \$225.18, \$172.64, \$183.82, \$238.24, \$179.27, \$194.38, \$203.92, \$210.74, and \$248.76. What was the average expenditure per month?

8. A farmer sold hogs weighing 324 lb., 332.5 lb., 384.75 lb., 336.25 lb., 395 lb., 347.5 lb., and 338 lb. What was the average weight per hog?

9. The pupils in an arithmetic class received marks of 78, 92, 84, 77, 86, 91, 100, 93, 75, 68, 81, 65, 87, 72, 60, 50, 87, 94, 76, 63, 79, 83, and 88. What was the average of the marks, to one decimal place?

10. If 18 hens consumed 100 pounds of feed in 40 days, what was the average amount consumed in one day by the whole flock, to one decimal place? What was the average amount consumed by one hen in one day?

11. After a family outing in an automobile, during which the car was driven 178.3 miles, the question was asked, "How many miles did we travel on one gallon of gasoline?" If 13.5 gallons of gasoline were used on the trip, what is the answer, to one decimal place?

12. A gasoline gas machine used 5.4 gallons of gasoline in one week. If the gasoline cost 37.5¢ per gallon, what was the cost per day, in cents, to one decimal place?

13. A gasoline lamp burned in 18 hours one gallon of gasoline, costing 24.2¢ per gallon. What was the cost in cents, to one decimal place, for gasoline for one hour?

14. The Hudson River was free from ice the following numbers of days in recent years: 259, 278, 267, 288, 318, 288, 268, 283, 271, 253, 260, 254, 244, 263, and 266. What was the average number of days in a year that the river was free from ice?

15. If a train takes 4.25 hours to go 140 miles, how many miles per hour does it travel, getting the result to one decimal place?

16. A farmer harvested 525 bushels of wheat from 14 acres of land. What was the average yield per acre, to one decimal place?

17. A neighboring farmer harvested 665 bushels of wheat from 20 acres. What was his average yield per acre?

18. A farmer who raised high grade hogs sold 45 hogs for $24,000. What was the average price received per hog, in dollars and cents?

19. The standing of a baseball team is found by dividing the number of games won by the number played. The result is expressed in thousandths. The team that made the highest standing between 1878 and 1917 was the Chicago team of 1880. That team won 67 games and lost 17 games. What was its standing?

20. If you have a football team, or any other athletic team that has played a number of games this year, compute its standing as explained in Example 19.

EXERCISE 26

The Great Lakes of the United States

	Superior	Michigan	Huron	Erie	Ontario
Greatest length in miles .	360	320	240	250	191
Greatest breadth in miles	160	85	101	57	53
Deepest soundings in feet	1,008	870	750	210	738
Area in square miles . .	32,060	22,300	23,000	10,000	7200

1. What is the total length of the Great Lakes in miles?

2. What is the average length in miles, to one decimal place?

3. What is the average of the greatest breadths of the lakes?

4. What is the average of the deepest soundings of the lakes?

5. What is the average of the areas of the lakes?

6. The longest lake is how many times as long as the shortest lake? (Get the result correct to one decimal place.)

7. The lake of greatest area is how many times as large as that of least area?

8. The deepest lake is how many times as deep as the shallowest lake?

9. The widest lake is how many times as wide as the narrowest lake?

21. Changing a common fraction to a decimal fraction.

Example 1. — Change $\frac{32}{125}$ to decimal form.

SOLUTION. — 1. $\frac{32}{125} = 32 \div 125$.
2. Therefore $\frac{32}{125} = .256$

$$
\begin{array}{r}
.256 \\
125\overline{)32.000} \\
250 \\
\hline
700 \\
625 \\
\hline
750 \\
750 \\
\hline
\end{array}
$$

In some cases, there is not an exact decimal equivalent for a given common fraction.

Example 2. — Express $\frac{246}{18}$ in decimal form, to two decimal places.

SOLUTION. — $\frac{246}{18} = 246 \div 18 = 13.66$

The result should be written either as 13.66+ or merely 13.66. The form $13.66\frac{2}{3}$ is not advisable.

Since the result to three places is 13.666, many give 13.67 as the result correct to two places. However, 13.66 is quite as satisfactory for all practical purposes.

$$
\begin{array}{r}
13.66 \\
18\overline{)246.00} \\
18 \\
\hline
66 \\
54 \\
\hline
120 \\
108 \\
\hline
120 \\
108 \\
\hline
12
\end{array}
$$

EXERCISE 27

Change the following fractions and mixed numbers to decimal form, giving the results to three decimal places:

1. $\frac{3}{5}$ **2.** $\frac{5}{8}$ **3.** $\frac{7}{25}$ **4.** $\frac{11}{40}$ **5.** $\frac{13}{20}$

6. $\frac{27}{50}$	9. $\frac{2}{3}$	12. $\frac{5}{12}$	15. $7\frac{3}{3}$	18. $6\frac{3}{4}$
7. $\frac{5}{6}$	10. $\frac{16}{4}$	13. $\frac{3}{16}$	16. $11\frac{4}{5}$	19. $27\frac{1}{6}$
8. $\frac{7}{8}$	11. $\frac{3}{11}$	14. $\frac{5}{32}$	17. $5\frac{4}{8}$	20. $19\frac{11}{12}$

In the following examples, first express the numbers in decimal form and then perform the operations indicated by the signs:

21. $11\frac{1}{2}+16\frac{3}{4}$	25. $2\frac{3}{5}\times11\frac{1}{2}$	29. $64\frac{5}{8}+72\frac{3}{4}$
22. $33\frac{1}{5}-16\frac{1}{2}$	26. $22\frac{3}{4}\div1\frac{1}{2}$	30. $45\frac{1}{6}-13\frac{2}{3}$
23. $6\frac{7}{25}\times3\frac{1}{2}$	27. $39\frac{7}{8}+23\frac{4}{5}$	31. $28\frac{3}{4}\times12\frac{1}{2}$
24. $15\frac{1}{8}-6\frac{3}{20}$	28. $16\frac{9}{20}-4\frac{1}{5}$	32. $17\frac{1}{8}\times19\frac{3}{5}$

EXERCISE 28

Journeys by Automobile

A man estimated that the engine of his car burns one gallon of gasoline, costing 24.2¢, in running 14 miles, and one quart of oil at 20¢ in going 60 miles; also that the average life of each of the tires is 5000 miles.

1. What is the expense per mile for gasoline? (Express the result in cents, to one decimal place.)

2. What is the expense per mile for oil?

3. What is the expense per mile for tires, if the tires cost $23.50 each?

4. What is the total of these ordinary running expenses of the car?

5. What does it cost this man for each of his daily trips to his office and home again, if the round trip is 11.3 miles?

6. The family, consisting of the man, his wife, two children over 5 years of age, and one under 5 years, went on an auto trip to a city, 155 miles distant. What was the cost for running expenses for the trip there?

7. The distance to that city by train is 140 miles. The fare on the train is 3¢ per mile for adults, and half fare for children under five. What would have been the railroad fare for the family had they gone by train?

8. If they had gone by the train, they would have had to use the street car line in their home town and also in the city. In their own town, the street car fare is 6¢ for adults and 3¢ for children under five. In the city, the fares are 7¢ for adults and 4¢ for children. What would have been the total of the street car fares?

9. What would have been the total cost for transportation had they gone on the railroad and street cars instead of in their own car?

10. *a.* A train takes $4\frac{3}{4}$ hours for the trip of 140 miles. How many miles does it average per hour, to one decimal place?

b. The trip by automobile took $7\frac{1}{2}$ hours of actual running time. How many miles per hour did the automobile average?

EXERCISE 29

Test on Decimals

1. From the sum of 3.24, 52.7, 8.435, and 6.902, subtract 45.003.

2. Find the product of 3.375 and 46.8.

3. Divide 77.8734 by 37.62.

4. Find $625 \times .004$ and divide the quotient by .125.

5. A workman receives 36.5¢ per hour. He worked 36 hours. How much pay should he get?

6. How high will a pile of 59 boards be if each board is .875 inches thick?

7. From a ten-yard bolt of cloth, a clerk sold one piece 3.25 yards in length and another piece 4.125 yards long. How much cloth should remain in the bolt?

8. A florist sold 243 plants for $461.60. What was the average price that he received per plant?

9. If a family burned one ton of coal in two weeks, how many pounds of coal did it burn per day? (Carry out the division to one decimal place.)

10. In one month, a boy sold 5 dozen eggs at 75¢ per dozen; 6 dozen eggs at 70¢ per dozen; and 9 dozen eggs at 65¢ per dozen. *a.* How much did he receive altogether for his eggs? *b.* What was the average price received per dozen eggs, expressed in cents to one decimal place?

EXERCISE 30

Miscellaneous Review

1. Reduce to their lowest terms:

$$\frac{2}{16},\ \frac{8}{32},\ \frac{4}{64},\ \frac{15}{45},\ \frac{75}{90},\ \frac{42}{120}$$

2. From the sum of $3786, $9245, $865, $727.50, $9346.25, $8294.75, and $1834.46, subtract $3768.29.

3. A man took 12 minutes to travel 5 miles. How many miles would he travel in one hour if he continued at the same rate?

4. A lake was completely frozen for the following numbers of days during recent years: 93 d., 110 d., 117 d., 95 d., 102 d., and 98 d. What is the average number of days that the lake was frozen?

5. Find the cost of seven loads of coal weighing 4275 lb., 4165 lb., 4425 lb., 4315 lb., 4260 lb., and 4235 lb. at $7.50 per ton.

6. What was the average weight per load in Example 5?

7. *a.* Find $13\frac{1}{2} \times 28\frac{3}{5}$. *b.* Find $16\frac{2}{3} \div 2\frac{1}{4}$.

8. Find the sum of $245\frac{1}{2}$, $316\frac{2}{3}$, $592\frac{3}{5}$, and $410\frac{3}{4}$.

EXERCISE 31

Cost of Linens, Towels, etc., for a Home

Use either the prices given in the following examples, or obtain prices from stores in your own community. An invoice or sales slip may be prepared for each or for some of these examples, if the teacher desires. If that is done, supply a date, a name and address for the store, and for the purchaser.

1. Find the cost of $\frac{3}{4}$ doz. sheets at \$2.55 per sheet, and $\frac{3}{4}$ doz. pairs of pillow slips at \$2.16 per pair.

2. Find the cost of $\frac{1}{2}$ doz. large towels at 96¢ each; $\frac{1}{2}$ doz. medium towels at 78¢ each; $\frac{1}{2}$ doz. wash cloths at 13¢ each; and $\frac{3}{4}$ doz. linen towels at 98¢ each.

3. Find the cost of $\frac{3}{4}$ doz. dish towels at 39¢ each; $\frac{1}{2}$ doz. glass towels at 42¢ each; and $\frac{1}{2}$ doz. dishcloths at 10¢ each.

4. Find the cost of 3 tablecloths at \$6.93, \$7.86, and \$10.92 respectively; one tablecloth at \$9.83; 3 lunch cloths at \$2.26, \$3.33, and \$5.40 respectively; $\frac{2}{3}$ doz. large napkins at \$6.75 per dozen; and $\frac{3}{4}$ doz. medium napkins at \$5.34 per dozen.

5. Find the total cost of the items listed in the previous examples.

IV. PERCENTAGE

22. Percentage is computing by hundredths. The symbol for per cent is %. 3 per cent is written 3%, and means $\frac{3}{100}$ or .03.

23. Writing per cents in decimal form.

Just as 3% means .03, so 18% means .18 and 90% means .90.

EXERCISE 32

Write the following in decimal form:

1. 7%	**6.** 24%	**11.** 85%	**16.** 10%	**21.** 60%
2. 9%	**7.** 19%	**12.** 72%	**17.** 40%	**22.** 16%
3. 5%	**8.** 1%	**13.** 91%	**18.** 70%	**23.** 66%
4. 2%	**9.** 33%	**14.** 63%	**19.** 15%	**24.** 80%
5. 11%	**10.** 25%	**15.** 13%	**20.** 35%	**25.** 75%

Write the following as per cents:

26. .17	**28.** .65	**30.** .24	**32.** .55	**34.** .43
27. .28	**29.** .49	**31.** .72	**33.** .98	**35.** .86

24. Using per cents in decimal form.

Example 1. — Just as $\frac{1}{2}$ of 50 means $\frac{1}{2} \times 50$ or 25, so 10% of 50 means $.10 \times 50$ or 5.

$$\begin{array}{r} 50 \\ .10 \\ \hline 5.00 \end{array}$$

Example 2. — Find 3% of 425.

3% of 425 = .03 × 425 = 12.75

$$\begin{array}{r} 425 \\ .03 \\ \hline 12.75 \end{array}$$

3% is called the *rate per cent*, 425 is called the *base*, and 12.75 is called the *percentage*.

25. The **rate per cent** is the number of hundredths to be found.

The **base** is the number of which a certain per cent is to be found.

The **percentage** is the result obtained by taking a certain per cent of the base.

Rule. — To find the percentage, multiply the base by the rate.

At this time, problems are stated in the form:

Find some per cent of some number.

The number of per cent mentioned is the rate per cent.

The number actually given or the number referred to immediately after the words " per cent of " is the base.

EXERCISE 33

1. Find 2% of 550; of 750; of 1200; of 8300; and of 6750.
2. Find 7% of each of the numbers in Example 1.
3. Find 8% of 650; of 3200; of 5300; and of 18,200.
4. Find 10% of each of the numbers in Example 3.
5. Find 15% of 349; of 5600; of 285; and of 970.
6. Find 6% of $27; of $48; of $125; and of $250.
7. Find 5% of $325; of $416; of $38; and of $63.
8. Find 11% of 84; of 75; of 98; and of 133.
9. Find 19% of 756.
10. Find 27% of 382.
11. Find 56% of 974.
12. Find 83% of $2350.

13. Find 16% of $275.50.
14. Find 31% of $185.50.
15. Find 4% of $62.75.
16. Find 9% of 38.45.
17. Find 13% of 26.4.
18. Find 83% of 100.
19. Find 57% of 1000.
20. Find 91% of 10,000.

21. State the rule for finding the percentage when the base and the rate are known.

22. How many are absent from a school of 450 pupils when 10% are absent?

23. A junior high school class in arithmetic consists of 35 pupils. 40% of these pupils are girls and 60% are boys. How many girls and how many boys are there in the class?

24. In one school of 800 pupils 5% were absent; in a second school of 500, 7% were absent. Which school had the greater number of pupils absent?

25. A florist planted 800 geranium slips; 13% of them failed to take root. How many did not take root, and how many did?

26. Out of 1250 trees planted in an orchard, 20% failed to grow. How many failed to grow?

27. A man agreed to give a real estate agent 5% of the amount for which the agent might sell the man's home. If the agent sold the home for $8350, how much must the man give the agent?

28. An agent was to receive 15% of the amount of all the money he collected as subscriptions for a certain magazine. He obtained $1375 in subscriptions. How much should he receive for himself?

29. The first payment on the Fourth Liberty Loan was to be 10% of the amount of the subscription. How much did a person have to pay who subscribed for:

a. A \$50 bond? *b.* A \$1000 bond? *c.* \$300 worth of bonds?

30. One day 20% of a class of 30 pupils were absent on account of a storm. How many were absent?

31. Automobile insurance companies insure a new automobile costing between \$1400 and \$2500 for 90% of its cost price. How much insurance can a man obtain on a car that cost him \$2150?

32. A contractor often charges 10% of the actual cost of a building as his fee for superintending the construction of the building. What would be the contractor's fee on a building that cost \$7258 for actual construction? With this fee, what would be the total cost of the building?

33. If a cow's milk contains about 4% of butter fat, how many pounds of butter does a cow produce that gives 4275 pounds of milk during the year?

34. A certain firm in Chicago agreed to give each of its employees at Christmas time 15% of the previous year's salary. How much did a person receive whose salary had been:

a. \$755? *b.* \$1425? *c.* \$1850? *d.* \$2400?

35. A certain mixture of feeds for chickens contains about 14% of protein, an essential food for egg production. How many pounds of protein are there in one hundred pounds of the mixture?

36. Architects formerly charged 5% of the cost of a dwelling as their fee for making the drawings and specifications for the building, and for superintending the construction of it. What was the fee for a dwelling that cost \$6740?

37. 65% of brass is copper and 35% is zinc. How much copper and how much zinc are there in a ton of brass?

38. In 1910, the population of the United States was approximately 91,970,000. Of these, about 46% lived in cities and towns, and the balance lived in the country. How many lived in cities and towns, and how many in the country?

39. A man had a road built by a contractor. The contractor included in his statement the following items:

Stone, $87.50; teaming, $35.75; labor, $72.50. He added to the total of these items a charge of 10% of their sum as his fee for superintending the work. What was the total cost of the road?

40. The total number of men in the U. S. Army during the war with Germany was about 4,000,000. Of these, 10% were national guardsmen, 13% were regular army men, and 77% were national army men. How many men were there in each branch of the army?

26. Writing per cents as common fractions.

Many per cents may be expressed conveniently as common fractions.

Thus, $10\% = \frac{10}{100} = \frac{1}{10}$.

EXERCISE 34

Write as common fractions in their lowest terms the following:

1. 50%	6. 80%	11. 5%	16. 2%	21. 12%
2. 20%	7. 30%	12. 35%	17. 22%	22. 28%
3. 40%	8. 70%	13. 85%	18. 4%	23. 65%
4. 90%	9. 25%	14. 15%	19. 16%	24. 1%
5. 60%	10. 75%	15. 45%	20. 32%	25. 64%

27. Memorizing certain per cents as common fractions.

Example 1. — $\frac{1}{4} = \frac{1}{4}$ of $\frac{100}{100} = \frac{25}{100} = 25\%$.

Example 2. — $\frac{3}{4} = 3 \times 25\% = 75\%$.

Example 3. — $\frac{1}{3} = \frac{1}{3}$ of $\frac{100}{100} = \frac{33\frac{1}{3}}{100} = 33\frac{1}{3}\%$.

EXERCISE 35

Express the following fractions as per cents. Memorize the first eight and as many more of the first twenty as you can.

1. $\frac{1}{2}$	6. $\frac{1}{8}$	11. $\frac{3}{4}$	16. $\frac{4}{5}$	21. $\frac{4}{8}$
2. $\frac{1}{3}$	7. $\frac{1}{10}$	12. $\frac{2}{5}$	17. $\frac{7}{8}$	22. $\frac{3}{12}$
3. $\frac{1}{4}$	8. $\frac{1}{12}$	13. $\frac{5}{6}$	18. $\frac{3}{5}$	23. $\frac{3}{6}$
4. $\frac{1}{5}$	9. $\frac{1}{16}$	14. $\frac{3}{8}$	19. $\frac{6}{8}$	24. $\frac{4}{20}$
5. $\frac{1}{6}$	10. $\frac{2}{3}$	15. $\frac{5}{8}$	20. $\frac{4}{6}$	25. $\frac{7}{28}$

Can you answer the following from memory?

26. $50\% = ?$	31. $3\frac{1}{3}\% = ?$	36. $37\frac{1}{2}\% = ?$
27. $16\frac{2}{3}\% = ?$	32. $12\frac{1}{2}\% = ?$	37. $\frac{5}{6} = ?\%$
28. $25\% = ?$	33. $\frac{1}{6} = ?\%$	38. $75\% = ?$
29. $\frac{1}{8} = ?\%$	34. $87\frac{1}{2}\% = ?$	39. $66\frac{2}{3}\% = ?$
30. $\frac{1}{5} = ?\%$	35. $60\% = ?$	40. $8\frac{1}{3}\% = ?$

28. Using per cents in common fraction form.

Example. — What is $87\frac{1}{2}\%$ of \$520?

SOLUTION. — $87\frac{1}{2}\%$ of $\$520 = \frac{7}{\cancel{8}_{1}} \times \overset{65}{\cancel{\$520}} = \$455$.

EXERCISE 36

In the following examples, use the per cent in common fraction form. Do as many as possible without pencil.

1. Find 50% of 180; of 3200; of 750; of 6400; of 260.
2. Find 25% of 800; of 2000; of \$684; of \$24.60.
3. Find 10% of \$950; of \$23.60; of \$25; of 24; of 16.
4. Find 20% of \$15; of \$45; of \$750; of \$16.50.
5. Find 75% of 40; of 200; of 1600; of 300 lb.; of 24 tons.
6. Find 40% of 25 mi.; of 65 bu.; of \$35.50; of 180 days.
7. Find $12\frac{1}{2}$% of \$80; of \$400; of \$720; of 24; of 36.
8. Find $16\frac{2}{3}$% of 360 mi.; of 900 ft.; of 1500 acres; of \$30.
9. Find $66\frac{2}{3}$% of \$1800; of 7590; of \$369.30; of 23.4.
10. Find $37\frac{1}{2}$% of \$640; of 5600; of 7280; of 16.4.
11. Find $33\frac{1}{3}$% of 150 days; of 3360; of \$132.66.
12. Find $87\frac{1}{2}$% of 8240; of \$45.60; of 24,000 sq. ft.
13. Find 60% of \$375.25; of 15 mi.; of 627.455.
14. 25% of a certain farm of 360 acres is wooded. How many acres are wooded?
15. 75% of one man's crop of apples was inferior because he had failed to properly spray them. If he had 1660 bushels, how many bushels were inferior?
16. In a certain month of 30 days, only 20% of the days were clear. How many days were clear and how many were cloudy?
17. A poultry grower found that about 40% of his flock were hens. If he had 1800 chickens in the flock, how many were hens?

18. The employees of a certain factory were given an increase in pay of $16\frac{2}{3}\%$ of their former wages. How much did the increase amount to for a person who had been getting:

a. \$3.00 per day? *b.* \$4.25 per day? *c.* \$5.40 per day?

19. Below are given the approximate per cent of increase in population in certain cities between 1900 and 1910. This increase is, approximately, what fractional part of the population in 1900?

Colorado Springs, Col.	37.5%	Everett, Mass.	37.5%
Decatur, Ill.	50%	Lexington, Ken.	$33\frac{1}{3}\%$
Madison, Wis.	$3\frac{1}{3}\%$	Portland, Me.	$16\frac{2}{3}\%$
Roanoke, Va.	$62\frac{1}{2}\%$	Shreveport, La.	75%
Somerville, Mass.	25%	Yonkers, N.Y.	$66\frac{2}{3}\%$

20. The land area of the United States in 1910 was about 1,903,290,000 acres. Of this amount 25% was improved as farms. How many acres of farm land was there?

21. In 1910, there were in the United States about 12,944,528 foreign-born residents. Of these, about $12\frac{1}{2}\%$ were illiterate. How many were illiterate?

22. The pay of a Lieutenant-Colonel in the United States army is $87\frac{1}{2}\%$ of that of a Colonel; that of a Major is 75% of that of a Colonel; that of a Captain is 60% of that of a Colonel; that of a First Lieutenant is $66\frac{2}{3}\%$ of that of a Major; that of a Second Lieutenant is \$50 less than 50% of the pay of the Lieutenant-Colonel. If the pay of a Colonel is \$4000 per year, what is the pay of each of the other officers?

23. In the national baseball championship series of 1919, the Cincinnati National League team won $62\frac{1}{2}\%$ of the games, and the Chicago American League team won

$37\frac{1}{2}\%$ of the games. If 8 games were played, how many did each team win?

24. *a.* Out of 8 football games played by the Yale team in 1919, the team won 75%. How many did it win? What per cent did it lose?

b. The Ohio State team played 6 games and won $83\frac{1}{3}\%$ of them. How many did it win?

c. The Pennsylvania team played 6 games. It won 50% of them; it lost $33\frac{1}{3}\%$ of them; and it tied $16\frac{2}{3}\%$ of them. How many did it win, lose, and tie?

25. A newsboy, who is selling papers for 2¢ each, makes a profit of 50% of the selling price. How much profit does he make on each paper?

29. Per cents equal to or more than 100%.

Example 1. — $100\% = \frac{100}{100} = 1$.

Then 100% of \$575 $= 1 \times \$575 = \$575$.

Example 2. — $225\% = \frac{225}{100} = 2\frac{1}{4}$.

Then 225% of \$360 $= 2\frac{1}{4} \times \$360$

$$= \frac{9}{\cancel{4}_{1}} \times \$\cancel{360}^{90} = \$810.$$

Example 3. — 314% of \$685 is how much?

$314\% = \frac{314}{100} = 3.14$.

Then 314% of \$685 $= 3.14 \times \$685 = \$2150.90$.

$$\begin{array}{r} \$685 \\ 3.14 \\ \hline 2740 \\ 685 \\ 2055 \\ \hline 2150.90 \end{array}$$

Since per cent means hundredths, any number of per cent can be written as a decimal by moving the decimal point of the number two places to the left.

Thus $425\% = 4.25$.

EXERCISE 37

How many times a number is:

1. 200% of it?
2. 500% of it?
3. 300% of it?
4. 700% of it?
5. 1000% of it?
6. 800% of it?
7. 150% of it?
8. 325% of it?
9. 475% of it?
10. 510% of it?
11. 820% of it?
12. $933\frac{1}{3}$% of it?

Express in decimal form:

13. 315%
14. 273%
15. 642%
16. 518%
17. 113%
18. 627%
19. 145%
20. 216%
21. 832%
22. 549%

How many per cent of a number is:

23. Five times the number? Ten times the number?
24. Seven times the number? Nine times the number?
25. Six times the number? Eleven times the number?
26. Four and one half times the number?
27. Two and one fourth times the number?
28. Three and one eighth times the number?
29. One and one third times the number?
30. Five and three fourths times the number?
31. Six and two thirds times the number?
32. One and three eighths times the number?
33. Two and five eighths times the number?
34. Three and one sixth times the number?
35. Four and five sixths times the number?

30. Using large per cents.

EXERCISE 38

1. In a percentage problem, what is the base? How can you usually pick out the number that is the base? (See § 25, p. 49.)

2. What does 19% mean? 123%?

3. What is the rule for finding the percentage when the base and the rate are known?

4. Certain merchants were reported as having made profits of 200% to 2000% on the money they had invested in their business during the year 1918. How many times their invested money was this?

5. The attendance in a certain school in 1919 was 350% of the attendance in that school in 1914. The attendance in 1914 was about 450 pupils. What was the attendance in 1919?

6. $100 invested in a certain company in 1908 had grown to 2800% of itself in 1919. How much was the amount in 1919?

7. A certain make of shoes cost in 1919 about 225% as much as they did in 1912. If they cost $5.00 per pair in 1912, what did they cost in 1919?

8. Anthracite coal that cost $9 per ton in 1914 cost 150% of that amount in 1919. What did it cost in 1919?

9. The average retail price of butter in 1909 was 34.9¢ per pound. In 1919, the price was about 183% of the price in 1909. What was the average price in 1919?

10. In 1910, the average price paid for milk in Chicago was 7.9¢ per quart. In 1919, the price was about 190% of that in 1910. What was the price in 1919?

11. In 1907, the cost of a pound of sugar in Denver was 6¢. In 1916 it was $133\frac{1}{3}\%$ as much. What was the cost in 1916?

12. In 1907, the average cost of one dozen fresh eggs in New York City was 32.4¢. In 1916, it was about 125% as much. What was the cost in 1916?

13. In 1907, the average cost of a one-eighth barrel sack of flour in Boston was \$.854. In 1916, it was about 140% as much. What was the cost in 1916?

14. In 1907, the average cost of a peck of potatoes in San Francisco was 36¢. In 1916, it was about 108% as much. What was the cost in 1916?

15. In 1907, the average cost of one pound of sirloin steak in Pittsburgh was 20.1¢. In 1916, it was about 144% as much. What was the cost in 1916?

31. Writing and using certain easy fractional per cents.

Example 1. — $\frac{1}{2}\%$ means $\frac{1}{2}$ of 1%.

Then $\frac{1}{2}\% = \frac{1}{2} \times .01 = .005$.

$$2\overline{)}.010$$
$$.005$$

Example 2. — $3\frac{1}{2}\%$ means $3\% + \frac{1}{2}\%$.

Then $3\frac{1}{2}\% = .03 + .005 = .035$.

Example 3. — $\frac{1}{4}\%$ means $\frac{1}{4}$ of 1%.

Then $\frac{1}{4}\% = \frac{1}{4} \times .01 = .0025$.

Then $3\frac{1}{4}\% = 3\% + \frac{1}{4}\% = .03 + .0025 = .0325$.

$$4\overline{)}.0100$$
$$.0025$$

Example 4. — Find $4\frac{1}{4}\%$ of \$2500.

Solution. — 1. $4\frac{1}{4}\% = .0425$.

2. Then $4\frac{1}{4}\%$ of \$2500 $= .0425 \times \$2500 = \$106.25$.

$$\begin{array}{r} \$2500 \\ .0425 \\ \hline 12500 \\ 5000 \\ 10000 \\ \hline \$106.2500 \end{array}$$

EXERCISE 39

Write the decimal value of:

1. $\frac{1}{2}\%$; $2\frac{1}{2}\%$; $3\frac{1}{2}\%$; $5\frac{1}{2}\%$.
2. $\frac{1}{4}\%$; $1\frac{1}{4}\%$; $3\frac{1}{4}\%$; $5\frac{1}{4}\%$.

3. $\frac{1}{5}\%$; $2\frac{1}{5}\%$; $3\frac{1}{5}\%$; $4\frac{1}{5}\%$.
4. $\frac{1}{8}\%$; $1\frac{1}{8}\%$; $3\frac{1}{8}\%$; $5\frac{1}{8}\%$.
5. $\frac{3}{4}\%$; $1\frac{3}{4}\%$; $3\frac{3}{4}\%$; $4\frac{3}{4}\%$.
6. $\frac{2}{5}\%$; $1\frac{2}{5}\%$; $4\frac{2}{5}\%$; $3\frac{2}{5}\%$.
7. $\frac{3}{8}\%$; $2\frac{3}{8}\%$; $4\frac{3}{8}\%$; $5\frac{3}{8}\%$.
8. $\frac{1}{10}\%$; $2\frac{1}{10}\%$; $3\frac{1}{10}\%$; $4\frac{1}{10}\%$.
9. $\frac{3}{10}\%$; $1\frac{3}{10}\%$; $5\frac{3}{10}\%$; $4\frac{3}{10}\%$.
10. $\frac{7}{10}\%$; $2\frac{7}{10}\%$; $3\frac{7}{10}\%$; $5\frac{7}{10}\%$.

Find:

11. $3\frac{1}{2}\%$ of \$6500
12. $4\frac{1}{2}\%$ of \$2745
13. $2\frac{1}{4}\%$ of \$3650
14. $4\frac{1}{4}\%$ of \$1150
15. $4\frac{3}{4}\%$ of \$4760
16. $1\frac{1}{8}\%$ of \$3250
17. $3\frac{2}{10}\%$ of \$6875
18. $4\frac{1}{4}\%$ of \$685
19. $1\frac{3}{4}\%$ of \$1525
20. $2\frac{1}{2}\%$ of \$775
21. $5\frac{1}{2}\%$ of \$5000
22. $3\frac{1}{2}\%$ of \$620
23. $2\frac{1}{4}\%$ of \$3500
24. $4\frac{3}{4}\%$ of \$1500
25. $4\frac{1}{4}\%$ of \$550

26. Some savings banks will pay to persons who leave their money on deposit in the bank for six months, $2\frac{1}{2}\%$ of the amount of the money on deposit. How much would you receive if you had \$125 on deposit?

27. In a certain ore, 52% is lead and $1\frac{3}{5}\%$ is silver. How much lead and how much silver are there in 10 tons of the ore?

28. Cloth loses about $1\frac{7}{8}\%$ of its length when it is sponged. What will be the loss of length in inches of a piece of cloth that is 12 yards long before it is sponged? How long will the piece be after it is sponged? (Change 12 yards to inches.)

29. Butter appears as tiny globules in milk. These globules are called "butter fat." The amount of butter

fat in the milk of one cow may be 4% of the weight of the milk produced by that cow. Other cows may produce milk that has more or less than that per cent of butter fat.

The records of the three best cows of a certain association of owners of cows is given below.

Cow Number	Total Number of Pounds of Milk in Year	Average % of Butter Fat in the Milk	Number of Pounds of Butter Fat	Value of the Butter Fat at 48¢ per Pound
1.	8109 lb.	4½%		
2.	5023 lb.	5.2%		
3.	4897 lb.	5.13%		

Complete the table by finding the number of pounds of butter fat produced by each cow and the value of that butter fat at 48¢ per pound.

30. The total population of the United States in 1910 was about 91,972,000. Of these, 7.1% lived in the New England States, 34.3% in the Atlantic States, 32.5% lived in the North Central States, 18.7% lived in the South Central States, 2.8% lived in the Mountain States, and 4.6% lived in the Pacific States. How many lived in each of the groups of states?

32. **The percentage formula.** — Review the definitions of base, rate per cent, and percentage as given in § 25, on page 49.

When the base and the rate are known, the percentage has been found by the rule *the base* × *the rate* = *the percentage.*

If B is allowed to represent the base, R the rate per cent, and P the percentage, this rule may be written

$$B \times R = P.$$

A rule expressed by letters that represent numbers is called a **formula**. The letters are connected by mathematical symbols that show what is to be done with the numbers.

Thus in the formula $B \times R = P$, the symbol $\times$ means that B is to be multiplied by R in order to get P.

Whenever you write the formula $B \times R = P$, think of the words represented by the letters and symbols.

33. Using the percentage formula.

Example 1. — Find P when $B = \$2785$ and $R = 5\%$.

Solution. — 1.	Since $B \times R = P$	$2785
2.	then $2785 \times .05 = P$	.05
3.	Therefore $\$139.25 = P$	$139.25

2785 in line 2 is put in place of B, and .05 is put in place of R. This is called **substituting their values for the known numbers** in the formula. (*B and R* are the known numbers; P is the unknown number to be found.)

Example 2. — Find P when $B = \$4836$ and $R = 16\frac{2}{3}\%$.

Solution. — 1.	Since $B \times R = P$	
2.	then $4836 \times \frac{1}{6} = P$	$(16\frac{2}{3}\% = \frac{1}{6})$
3.	$\$806 = P$	

EXERCISE 40

Using the form of solution illustrated above, find P:

Ex. No.	When	and	Ex. No.	When	and
1.	B = $7860	R = 8%	6.	R = 11%	B = 54,900
2.	B = $2745	R = 1%	7.	R = 5½%	B = $27,500
3.	B = $3750	R = 3½%	8.	B = $263.75	R = 9%
4.	B = 9280	R = 4½%	9.	B = $484.00	R = 12½%
5.	B = 368	R = 125%	10.	R = 4¼%	B = $5275

Using the same form of solution, find the percentage:

11. When the base is \$382.75 and the rate is 10%.
12. When the rate is 7% and the base is 194.
13. When the base is 625.8 and the rate is $33\frac{1}{3}$%.
14. When the rate is 75% and the base is 38.
15. When the rate is $37\frac{1}{2}$% and the base is \$4240.

EXERCISE 41

Miscellaneous Review Drill

1. Find the sums of the following columns. Time yourself.

a	*b*	*c*	*d*	*e*	*f*	*g*
978	495	673	472	286	389	274
185	674	397	274	186	497	635
324	579	354	983	168	325	657
486	547	398	268	541	385	589
327	587	693	264	217	485	431
986	873	289	895	400	46	68
35	507	780	409	900	607	306
290	861	547	689	999	748	637
321	542	368	437	548	974	379

2. Find $21\frac{2}{3} \times 2\frac{1}{2} \times 4\frac{1}{4}$.
3. Find $32\frac{1}{2} \div 6\frac{1}{4}$.
4. Find $12\frac{1}{8} + 16\frac{2}{3} - 11\frac{1}{6}$.
5. Find $14\frac{1}{5} + 26\frac{2}{3} - 16\frac{1}{2}$.
6. How much is 50% of:

a. one bushel? *b.* one ton? *c.* one gallon?
d. one acre? *e.* one mile? *f.* one day?

7. Repeat Example 6, substituting $12\frac{1}{2}$% for 50%.
8. Out of 22 tons of coal on hand on October 1st, a man found that he had used about 70% up to January 1st. How many tons did he have left on January 1st?

9. A certain firm sold $250,000 worth of goods in 1915. In 1918, its sales were 425% of that amount. What were its sales in 1918?

10. The average wage earned by the 284 employees of a certain factory was $1375.15 per year. How much did the company pay out in wages during the year?

11. Give the per cent equivalent to:

a. $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{8}$, $\frac{1}{10}$, $\frac{1}{12}$.

b. $3\frac{1}{2}$, $2\frac{1}{4}$, $5\frac{1}{2}$, $6\frac{1}{5}$, $2\frac{3}{4}$.

12. How many times a number is:

300% of it? 500% of it? 250% of it? 175% of it?

EXERCISE 42

THE PANAMA CANAL

TABLE A. — Below are the distances from New York City by the former all-water route and by the Panama Canal.

To	By Former Water Route	By Panama Canal
San Francisco	13,135 miles	5,262 miles
Hawaii	12,800 miles	7,000 miles
Manila	17,800 miles	12,000 miles

TABLE B. — Below are the numbers of vessels of certain nations that passed through the canal.

In	British	Chilian	Dutch	Danish	French	Japanese	Norwegian	United States
1915	465	35	7	23	3	6	42	470
1916	358	33	15	18	1	24	45	238
1917	780	90	74	43	9	72	150	464

TABLE C. — Below is given the record of the traffic on the canal during the first half of the year 1917.

	From Atlantic to Pacific	From Pacific to Atlantic
Total vessels	450	456
Net tonnage (carrying capacity)	1,445,154	1,436,552
Tons of cargo	1,438,625	2,103,942

1. How much less is the distance via the Panama Canal than by the former water route: *a.* To San Francisco? *b.* to Hawaii? *c.* to Manila?

2. What was the total number of vessels of the nations named that passed through the canal in each year?

3. What was the average weight of cargo in the first half of 1917:

a. From the Atlantic to the Pacific? *b.* From the Pacific to the Atlantic?

4. What was the average net tonnage per vessel:

a. From the Atlantic to the Pacific?

b. From the Pacific to the Atlantic?

5. If the charge for passage through the canal of a ship with cargo is $1.20 per ton of net tonnage (carrying capacity), what would be the charge for a vessel having the tonnage found in Example 4?

6. If the charge for passage through the canal of a ship without passengers is 40% less than when loaded, as in Example 5, what would be the charge for such a vessel, if the tonnage is that found in Example 4?

V. SOME APPLICATIONS OF PERCENTAGE

34. Application of percentage to increase and decrease.

By increasing a quantity or number is meant adding to it; by decreasing a quantity or number is meant subtracting from it.

Example 1. — How much is 350 increased by 10% of itself?

Solution. — 1. This means that 10% of "itself," that is, 10% of 350, is to be added to 350. 350 is the base. The percentage is the increase.

2. 10% of $350 = .10 \times 350 = 35$, the increase.

3. The result $= 350 + 35 = 385$.

Example 2. — How much is \$275 decreased by 15% of itself?

Solution. — 1. This means that 15% of \$275 is to be subtracted from \$275. The base = \$275 and the rate = 15%; the decrease = the percentage.

2. 15% of \$275 = .15 × 275 = \$41.25.

$$\begin{array}{r} \$275 \\ .15 \\ \hline 1375 \\ 275 \\ \hline \$41.25 \end{array}$$

3. The result = \$275 − \$41.25
= \$233.75.

$$\begin{array}{r} \$275.00 \\ 41.25 \\ \hline \$233.75 \end{array}$$

EXERCISE 43

Find:

1. 650 increased by 5% of itself.
2. 460 increased by 6% of itself.
3. 480 increased by $12\frac{1}{2}\%$ of itself.
4. 540 increased by 25% of itself.
5. 790 decreased by 20% of itself.
6. 840 increased by 50% of itself.
7. 360 decreased by $33\frac{1}{3}\%$ of itself.
8. 24 increased by 75% of itself.
9. 18 decreased by $16\frac{2}{3}\%$ of itself.
10. 32 increased by 11% of itself.
11. 85 decreased by 9% of itself.
12. 64 increased by 30% of itself.
13. 45 decreased by 10% of itself.
14. 86 increased by 5% of itself.

15. A grocer agreed to decrease the prices of articles in his store 5% of their marked prices if the purchaser paid cash. What would be the cash price of an article marked:

a. 69¢? *b.* 85¢? *c.* \$1.23? *d.* \$3.42? *e.* 64¢?

16. A merchant advertised that he was decreasing his prices 20% to bring in trade after the holidays. What would be the sale price of an article that had been marked:

a. \$5.45? *b.* \$9.75? *c.* \$3.15? *d.* \$1.85? *e.* \$1.89?

17. Owing to decreased hours of labor, the quantity of material produced in a certain factory decreased about 15%. If the factory formerly produced 2600 articles per day, how many is it now producing?

18. The workmen of a certain factory asked for an increase in their pay of 18% of their former wages. If the total wages paid formerly have been \$6745.50 weekly, what will the owners of the factory have to pay weekly, if they give the men the increase requested?

19. A certain kind of rug that sold for \$55 in 1910 had increased 200% of this price by the end of 1919. What was the selling price in 1919?

20. A new road was built in front of a man's property. He estimated that it increased the value of his property 15% of its former value. If he formerly asked \$8750 for it, what should he ask for it now?

21. The records of a certain school show that the attendance has increased each year about 7% of the attendance of the previous year. If the attendance now is 820, what will the attendance be, approximately, three years from now if the same rate of increase continues?

22. The value of a new automobile decreases at least 35% through use of the car during the first year after it is purchased. If a car costs \$1475 when new, what will its value be at the end of the first year?

23. In December, 1919, the selling prices of certain rugs were increased 8% of their former prices. What was the new price of a rug that had been marked:

a. \$165.00? *b.* \$125.00? *c.* \$75.00?

24. A dealer informed a customer that the prices of tires were to be increased 15% of their present prices, in two weeks. The customer needed in the near future one tire then marked \$23.70; one marked \$45.54; one tube marked \$3.20; and one tube marked \$5.40. What would he save by buying at once?

25. The average weight of a child four years old is 41 lb. *a.* By the age of eight, the child's weight increases about $37\frac{1}{2}\%$ of the weight at four. What is the approximate weight at eight?

b. By the age of twelve, the average weight increases about 95% of the weight at age four. What is the approximate weight at twelve?

26. Bring to class any problems of increase or decrease that you can get from acquaintances or from the newspapers or magazines.

35. Application of percentage to profit and loss. Merchants must add some amount to the cost of articles that they sell so that they will receive money with which to pay their clerks, their rent, their bills for heating, lighting, etc., and pay for their own services and ability. This amount is called the **gross profit.** Remember that many expenses must be paid out of the gross profit. That which remains after all expenses are paid is called the **net profit.**

The cost + the gross profit = the selling price.

Sometimes the merchant adds to the cost a certain per cent of the cost in order to get the selling price.

Thus, an article cost \$1.39. The merchant decides to add a gross profit of 35% of the cost.

35% of \$1.39 = .35 + \$1.39 = .4865 or \$.49

The selling price = \$1.39 + \$.49

= \$1.88

\$1.39
.35
695
417
\$.4865

Remember that the number coming after the words " per cent of " is the base, because the base is the number of which a certain per cent is to be found. The gross profit is the percentage.

EXERCISE 44

1. What was the selling price of an article bought for \$1.50 and sold at a gross profit of:

a. 50¢? *b.* 75¢? *c.* \$1.00? *d.* 80¢?

2. What was the gross profit if an article bought for \$1.25 was sold for:

a. \$1.45? *b.* \$1.60? *c.* \$2.00? *d.* \$1.85?

3. An article was bought for \$2.00. What was the net profit if the article was sold for \$3.00, and the expenses connected with the sale amounted to 25¢?

Determine the selling price:

4. Of articles marked to give a gross profit of 25% of the cost if the original cost was:

a. \$2.00; *b.* \$1.80; *c.* \$.88; *d.* \$.64; *e.* \$.39

5. Of articles marked to give a gross profit of 40% of the cost if the cost was:

a. 45¢; *b.* 75¢; *c.* 60¢; *d.* 18¢; *e.* 34¢

6. Of articles marked to give a gross gain of $33\frac{1}{3}\%$ of the cost if the cost was:

a. 39¢; *b.* \$1.20; *c.* 72¢; *d.* \$1.65; *e.* \$2.85

7. Of articles marked to give a gross profit of $12\frac{1}{2}\%$ of the cost if the cost was:

a. \$2.34; *b.* \$5.60; *c.* \$11.50; *d.* \$7.65; *e.* \$9.48

Find the selling price of an article if the cost was the amount given below and the gross profit was the rate per cent of the cost that stands beside the cost:

8. \$7.24; 35%
9. \$1250; 15%
10. \$250; 20%
11. \$5.84; 25%
12. \$9.62; 30%
13. \$11.48; 35%

14. \$.89; 45% 16. \$.93; 55% 18. \$2.47; 30%

15. \$.76; 50% 17. \$1.76; 45% 19. \$325.50; 25%

20. In all these examples, what number is used as the base? What is the percentage? Tell some of the expenses that must be paid out of the gross profits.

21. A man owned a house and lot that had cost him \$6250. In order to sell it quickly, he offered to sell it at a loss of 10% of the cost. What was he ready to accept for it?

22. A business man, compelled to obtain some cash quickly, had to sell at a loss of 5% of the cost some property that had cost him \$765.70. What did he receive for the property?

23. A farmer's barn and contents, valued at \$8500, was damaged by fire. The loss was estimated to be about 20% of the value of the property. What was the value after the fire?

24. A merchant has marked his goods so as to make a gross profit of 10% *of the selling price.* What did an article cost him which he sells for:

a. \$2.50? *b.* \$3.00? *c.* \$.80? *d.* \$50? *e.* \$.60?

25. Another merchant has marked his goods for sale so that his gross profits are 25% of the selling prices. What did articles cost him that he sells for:

a. \$5.00? *b.* \$3.60? *c.* \$2.75? *d.* \$.68? *e.* \$.48?

NOTE. — Just how a merchant may determine his prices so that he will make a gross profit of a certain per cent of the selling price will be taught in Book II.

36. Application of percentage to discount.

The price at which an article is marked or advertised for sale is called the **list price.** Sometimes the article is

sold for less than the list price. The amount of the reduction in price is called a **discount.**

A discount is usually a certain rate per cent of the list price.

Thus a discount of 5% means that 5% of the list price is to be subtracted from the list price.

Since the discount is a certain per cent of the list price, then the list price is the base, and the discount is the percentage.

Example. — What is the discount and the selling price of a coat marked for sale at $95, with a discount of 10%?

Solution. — 1. It is understood that the list price is the base.
2. Then the discount = 10% of $95 = $9.50.
3. Then the selling price = $95 − $9.50 = $85.50.

EXERCISE 45

1. Why do merchants give a discount?

2. When you know the list price and the rate of discount, how do you find the discount? How do you find the actual selling price?

3. Bring to class any examples about discount that you can get from the papers, from advertisements, or from catalogues.

Determine the discount and the selling price if:

4. The rate of discount is 10% and an article is listed at:

a. $1.80; *b.* $2.50; *c.* $3.45; *d.* $.80; *e.* $.50

5. The rate of discount is $33\frac{1}{3}$% and an article is listed at:

a. 84¢; *b.* $1.35; *c.* $2.70; *d.* $1.50; *e.* $7.50

6. The rate of discount is 25% and an article is listed at:

a. \$12.00; *b.* \$23.00; *c.* \$15.75; *d.* \$12.80; *e.* \$9.65

7. A clothing store advertised a discount of $16\frac{2}{3}\%$ on all suits and overcoats. What would be the sale price of a suit listed at \$65.00 and of an overcoat listed at \$70?

8. A merchant received an invoice (see § 5, p. 11) for goods bought from a wholesaler. The amount of the invoice was \$73.85. He was offered a discount of 3% if he paid for the goods within 10 days. How much would he save by paying the bill within the ten days?

9. A man bought a barrel containing $54\frac{1}{2}$ gallons of gasoline at $37\frac{1}{2}¢$ per gallon. He was offered a discount of 2% if he paid the bill in ten days.

a. If he paid the bill at once, how much should he have sent the company?

b. If the freight on the barrel was 84¢, and the cost of having the barrel brought to his house was \$1.50, what was the total cost of the $54\frac{1}{2}$ gallons of gasoline?

c. What was the cost per gallon, expressed in cents, correct to one decimal place?

10. A clerk in a department store bought for her own use the following articles: 2 pairs of stockings marked \$1.75 per pair; 1 pair of woolen mittens, marked \$.95; and $3\frac{1}{2}$ yards of lace at \$.65 per yard. Clerks in this store are given a discount of 10% of the list price, on articles bought for their own use. How much did the clerk have to pay for these articles?

11. A man acquainted with a wholesale dealer bought an article listed at \$3.80. He was given a discount of 50% of the list price. What did he pay for the article?

12. A company handling automobile supplies offered other dealers selling their goods a discount of 15% on an article that was listed at $60. What would an ordinary customer have to pay for this article? What would another dealer have to pay for it?

13. An agent who was just starting in business in a certain town offered a discount of 20% on an automobile battery that was listed at $45.50. What would the battery cost a purchaser? Why should a dealer offer the discount under these circumstances?

14. A man offered his lot for sale for $1750. He agreed to give a discount of $2\frac{1}{2}\%$ if the purchaser paid cash for the lot. How much would it cost the purchaser if he paid cash? Why does the owner offer this discount?

15. A merchant bought suits of clothes from the wholesale dealer for $37.50.

a. He marked them to sell so that his gross profit would be 40% of the cost. What was the price at which they were listed for sale?

b. At the close of the season, he advertised for sale a few that were left over, at a discount of 15% from his list price. What did he receive for *these* suits during the sale?

c. What was his gross profit on one of the suits sold during the sale?

37. Application of percentage to finding commissions. Firms and individuals are compelled often to hire agents to buy or sell goods for them, or to conduct their business for them. They often pay these agents a certain per cent of the money value of the property handled by the agent. This pay is then called a **commission.**

The **gross proceeds** of a sale is the sum received from the purchaser. The **net proceeds** is the amount that remains after the agent's commission and all other expenses have been subtracted from the gross proceeds.

EXERCISE 46

1. A boy solicited subscriptions for a magazine. His commission was to be 20% of the amount of the subscriptions that he collected.

a. If he collected $75 in subscriptions, how much was his commission?

b. How much were the net proceeds that he had to send to the company?

2. A college student sold during his summer vacation 525 books at $1.25 each. He received a commission of 25% of the price of the book.

a. What were the gross proceeds of the sales?

b. What was his commission?

3. An attorney charged a commission of 10% for collecting an unpaid bill of $325.

a. What was his commission?

b. If he also kept $15 which he had had to spend in collecting the money, what were the net proceeds received by his client?

4. A real estate agent sold a piece of property for $15,500. He charged a commission of 5% of the sale price.

a. What was his commission?

b. If other expenses in making the sale amounted to $25.35, what were the net proceeds received by the former owner of the property?

5. An insurance agent received as commission 15% of the money he collected from people who took out fire insurance policies through his office.

a. If the total amount collected through his office in one month amounted to \$1855.75, how much were his commissions during that month?

b. What were the net proceeds which he had to send to the fire insurance companies?

6. A commission merchant sold for a farmer 1525 bushels of corn at \$1.29 per bushel. He charged a commission of 2% of the gross proceeds, and also sent the farmer a bill for \$32.25 for other expenses.

a. How much should his commission have been?

b. What should the farmer have received?

7. A grain dealer sold for a farmer 3720 bushels of oats at 78¢ per bushel. He charged a commission of $1\frac{1}{2}$%, and included a bill of \$47.25 for other expenses. How much should the farmer have received?

8. An auctioneer was to receive a commission of 5% of the gross proceeds of a sale he was conducting for a farmer. If the gross proceeds were \$3247.50, how much should his commission have been?

9. How much should the owner of a lot receive from a real estate agent who sold his lot for \$1250, if the agent was to receive a commission of 5%?

10. A salesman for an automobile company received \$75.00 per month, and in addition, 2% of the sale price of each automobile that he sold. During the year, he sold 24 cars for \$1450 each.

a. What was the total amount of his commissions?

b. What was his total income for the year?

c. What was his average income per month?

11. A money lender charged a commission of $1\frac{1}{2}\%$ for obtaining a loan of \$4200 for a man. How much was his commission?

12. Bring to class any commission problems that you can get from your parents, or from business men in your town.

EXERCISE 47

Miscellaneous Abstract Drill

1. Find $13\frac{3}{5}+26\frac{1}{3}+18\frac{1}{2}$.
2. Find $16\frac{2}{3}\times 4\frac{2}{5}$.
3. Find $28\frac{1}{4}\times 5\frac{1}{2}$.
4. Find $21\frac{3}{5}\div 2\frac{1}{10}$.
5. Find $75\frac{1}{8}\div 3\frac{1}{3}$.
6. Find 26.4×3.1416; also find $423.8\times .7854$.
7. Divide to two decimal places:

a. $1832.4\div 62.5$ *b.* $379.58\div 41.163$

8. Write the per cent equal to:

$$\frac{3}{6},\ \frac{5}{8},\ \frac{4}{12},\ \frac{7}{35},\ \frac{8}{48},\ \frac{9}{10}.$$

9. Write in decimal form: 325%, 614%, $2\frac{1}{2}\%$, $4\frac{1}{2}\%$.
10. Find $3\frac{1}{2}\%$ of \$255; $4\frac{1}{4}\%$ of \$300; and $4\frac{1}{2}\%$ of \$750.

EXERCISE 48

Miscellaneous Review

1. A fruit dealer purchased 125 boxes of oranges at \$2.40 per box. He marked them so that his gross profit would be $12\frac{1}{2}\%$ of the cost. At what price were they marked? What were his gross profits on each box and upon the 125 boxes?

2. Superintendents whose schools employed a total of 238,573 teachers in 1918–1919 reported that about 22%

of these teachers had dropped out at the end of the year. How many dropped out?

3. There are required for the schools of the United States at the present time (1919) about 650,000 teachers. During the year 1919–1920, about 6% of this number could not be obtained, and about 10% were teachers with insufficient training. How many unfilled positions were there, and how many poorly trained teachers were there?

4. A grocer bought eggs from farmers at 60¢ per dozen. He marked them so that his gross profit would be $16\frac{2}{3}\%$ of the cost. What were his gross profits on 72 dozen eggs?

5. Study of the average expenditures of families has shown that about 43% of the family income is spent for food, about 18% for rent, about 13% for clothing, about $5\frac{1}{2}\%$ for fuel and light, and about $20\frac{1}{2}\%$ for other items. Determine the amounts that a family having an income of $1800 per year should spend if they adjust their living expenses according to these averages.

6. What was the cash cost of a $7.50 coffee percolator, a $1.25 pair of shears, and a 35¢ corn popper bought at a hardware store that gives a 3% discount from the list prices when the purchaser pays cash?

7. The 58th Congress of the United States (1905–1906) appropriated about $1,600,000,000. The 63d Congress (1915–1916) appropriated a little less than 140% as much as the 58th Congress. How much, approximately, did the 63d Congress appropriate?

8. A wholesale dealer bought 3 tons of sugar at $18.00 per hundred pounds. He sold it so that he obtained a gross profit of $5\frac{1}{2}\%$ of the cost.

a. How much did he charge for it per hundred pounds?

b. What were his gross profits?

9. A book dealer sold 33 school books at 90¢ each. If he bought them at a discount of 20% of this list price, what did he pay for the books, and what were his gross profits?

10. The value of farm land in a certain neighborhood has increased about 40% in the last five years. One man bought a farm five years ago for $175 per acre. What should be its value per acre to-day?

11. If you know the cost of an article and the rate per cent of gross profit, how do you find the selling price?

12. If you know the price at which an article is marked, and the rate per cent of discount, how do you find the actual selling price?

13. What is meant by a commission?

14. A dry goods store advertised a discount of 15% on silks during its January sale. What would $4\frac{1}{2}$ yards of silk cost that had been marked $3.50 per yard?

15. The same store advertised a discount of $12\frac{1}{2}$% on towels, sheets, and pillow casings. Determine the cost during the sale of six towels marked $1.25 each, four sheets marked $2.50 each, and three pair of pillow casings marked $2.65 per pair.

VI. PERCENTAGE (*Continued*)

Finding the Base and the Rate Per Cent

38. Finding the rate per cent when the base and the percentage are known.

Development. — 1. What is the rule for finding the percentage when the base and the rate are known? Notice then that the percentage is the product of the base and the rate.

2. If $25 \times$ some number $= 75$, then the number $= 75 \div 25 = 3$.
If $25 \times$ some number $= 20$, then the number $= 20 \div 25 = \frac{4}{5}$.
If $25 \times$ some rate % $= 5$, then the rate % $= 5 \div 25 = \frac{1}{5}$
$= 20\%$.
If $32 \times$ some rate % $= 4$, then the rate % $= 4 \div 32 = \frac{1}{8}$
$= 12\frac{1}{2}\%$.

In the last example, the percentage, 4, divided by the base, 32, gives the rate, $12\frac{1}{2}\%$.

Since the percentage is always the product of the base and the rate, then the result of dividing the percentage by the base must give the rate.

Rule. — To find the rate per cent when the base and the percentage are known, divide the percentage by the base.

The formula is: $R = P \div B$.

Example 1. — What per cent of 176 is 22?

Solution. — 1. The base, B, is 176, since it follows the words "per cent of." The percentage, P, is 22, since the example states that it is some per cent of 176. R is to be found.

2. The formula is: $R = P \div B$.
3. Then the rate $= 22 \div 176 = \frac{22}{176}$
$= \frac{1}{8} = 12\frac{1}{2}\%$

NOTE. — 1. In many problems, it is wise to form the fraction whose numerator is the percentage and whose denominator is the base. This fraction will often reduce to one of the fractions whose equal per cent is known from § 27. This is particularly true when the numbers are small, for then the problem can often be solved mentally.

2. Remember that the base comes after "per cent of," and that the percentage is the subject of the sentence.

Example 2. — 18.9 is what per cent of 540?

Solution. — 1. $P = 18.9$; $B = 540$; and $R = ?$
2. The formula is: $R = P \div B$.
3. Then the rate $= 18.9 \div 540$
$= .035$
$= 3.5\%$

$$540\overline{)18.900}\quad .035$$
16 20
2 700
2 700

After obtaining the rate as a decimal, the equal rate per cent is found by moving the decimal point two places to the right. Review if necessary § 29 and § 30.

Thus, $.0225 = 2.25\%$; $.054 = 5.4\%$; $1.2 = 120\%$.

Example 3. — What per cent of 72 is 90?

Solution. — 1. $P = 90$; $B = 72$; $R = ?$
2. The formula is: $R = P \div B$.
3. Then the rate $= 90 \div 72$
$= 1.25$
$= 125\%$

$$72\overline{)90.00}\quad 1.25$$
72
180
144
360
360

NOTE. — Do not think that the percentage is always the smaller number, and the base the larger number. In this example, the percentage is the larger and the base is the smaller.

EXERCISE 49

1. What per cent of 30 is 10?
2. 6 is what per cent of 48?
3. What per cent of 20 is 10?
4. 4 is what per cent of 20?
5. What per cent of 20 is 5?
6. What per cent of 20 is 2?
7. 6 is what per cent of 24?
8. 4 is what per cent of 24?
9. 8 is what per cent of 24?
10. What per cent of 24 is 3?
11. What per cent of 32 is 16?
12. 8 is what per cent of 32?
13. 20 is what per cent of 32?
14. What per cent of 48 is 12?
15. What per cent of 60 is 40?
16. 16 is what per cent of 48?
17. 12 is what per cent of 60?
18. 5 is what per cent of 25?
19. 9 is what per cent of 27?
20. What per cent of 54 is 36?
21. What per cent of 72 is 27?
22. 12 is what per cent of 72?
23. 35 is what per cent of 70?
24. What per cent of 105 is 15?
25. 9 is what per cent of 3?
26. What per cent of 5 is 10?
27. What per cent of 4 is 22?
28. 35 is what per cent of 15?

29. 42 is what per cent of 10?

30. What per cent of 16 is 50?

31. What per cent of 280 is 14? is 19.6?

32. What per cent of \$564 is \$16.92? is \$39.48?

33. \$3.75 is what per cent of \$75? of \$375?

34. What per cent of 240 miles is 30 miles? is 80 miles?

35. \$125 is what per cent of \$5000?

36. What per cent of a month is a week?

37. What per cent of a ton is a hundredweight?

38. What per cent of a bushel is a dry quart?

39. A liquid quart is what per cent of a gallon?

40. A pint is what per cent of a gallon?

41. In one class of 30 pupils, 5 pupils failed to do satisfactory work. In a second class that had 35 pupils, 7 failed to do satisfactory work. Which class had the greater per cent of unsatisfactory pupils?

42. On an examination, 10 pupils in a class of 25 made marks above 90. What per cent of the class were they? In another class, having 24 pupils, 9 pupils received marks above 90. What per cent were they? Which class had the greater per cent of high-grade pupils?

43. *a.* In a class of 35 pupils, 21 are boys. What per cent of the class are boys and what per cent are girls?

b. In this school, there are 540 pupils, of whom 297 are boys. What per cent of the pupils in the school are boys?

c. How does the per cent of boys in the class compare with the per cent of boys in the school?

44. In a junior-senior high school of about 300 pupils, 36 pupils were in the lowest or seventh-grade class. What per cent of the pupils in the school were in the lowest class? What per cent were in the other classes?

45. In a flock of 65 young chickens, 26 were pullets. What per cent were pullets?

46. One boy placed 75 eggs in his incubator. 50 chicks were hatched. What per cent of the eggs "hatched out"?

47. A neighboring boy placed 120 eggs in his incubator. He got 90 chicks.

a. How many per cent of his eggs hatched out?

b. Which boy had the better success?

48. A family spent \$300 per year for rent out of an income of \$1500. What per cent of their income was spent for rent?

49. The same family spent \$60 per year for coal. What per cent of their income did they spend for coal?

50. They spent \$48 per year for electricity. What per cent of their income did they spend for electricity?

EXERCISE 50

In the following examples, you are to find either the percentage or the rate. Use the form of solution given in Example 3, p. 81.

1. Find R when $P = \$81.00$ and $B = \$2700$.
2. Find P when $B = \$500$ and $R = 8\%$.
3. Find R when $B = 5280$ ft. and $P = 1320$ ft.
4. Find P when $R = 4.5\%$ and $B = 3880$.
5. Find R when $P = \$36.54$ and $B = \$32.48$.
6. Find P when $B = 426.9$ and $R = 16\frac{2}{3}\%$.
7. Find R when $B = \$5400$ and $P = \$8100$.
8. Find R when $P = 863.04$ and $B = \$647.28$.
9. Find P when $B = \$725.25$ and $R = 125\%$.
10. Find R when $B = \$9842$ and $P = \$9349.90$.

39. Application of finding the rate to problems of increase and decrease.

Example 1. — A man, whose salary had been \$120 per month, had his pay increased to \$150 per month. What per cent of his former salary was the increase in his salary?

Solution. — 1. B = his former salary, \$120; P = the increase, \$30; R = ?

2. The formula is: $R = P \div B$.

3. Then the rate of increase = \$30 ÷ \$120 = $\frac{1}{4}$ = 25%.

Example 2. — In 1907, the average price per pound of butter was 33.1¢. In 1916, it was 39.1¢. What per cent of the cost in 1907 was the increase?

Solution. — 1. B = cost in 1907 = 33.1¢; P = the increase = 6¢; R = ?

2. The formula is: $R = P \div B$.

3. Then the rate of increase = 6 ÷ 33.1

= .181+ = 18.1%.

39.1
33.1
6.0

NOTE. — Carry out the division to three or four decimal places, if it is not an exact quotient. This will give the result correct to tenths or hundredths of a per cent.

EXERCISE 51

Increased Wages and Prices in One Locality

1. In 1907, bricklayers were paid 62.5¢ per hour. In 1919, they were paid \$1.00 per hour. The increase was what per cent of the pay in 1907?

2. In 1907, plasterers' helpers received 40¢ per hour; in 1916, they received 50¢ per hour. What per cent of the pay in 1907 was the increase?

3. *a.* In 1907, plumbers received 62.5¢ per hour. In 1916, they received 75¢ per hour. What was the rate per cent of increase from 1907 to 1916?

b. In 1919, they received $1.00 per hour. The increase from 1907 to 1919 was what per cent of the pay in 1907?

4. *a.* In 1907, painters received 50¢ per hour and in 1916, they received 60¢ per hour. What per cent of the pay in 1907 was the increase?

b. In 1919, they received $1.00 per hour. What per cent of the pay in 1907 was the increase between then and 1919?

c. What per cent of the pay in 1916 was the increase between then and 1919?

5. Teamsters received $4.50 per day in 1914 and $7.50 per day in 1919.

a. What per cent of their pay in 1914 was the increase?

b. What per cent of their pay in 1914 was their pay in 1919?

6. Laborers received $2.25 per day in 1917, and $3.00 per day in 1918.

a. What per cent of their pay in 1917 was the increase?

b. Many laborers received $5.00 per day in 1919. What per cent of their pay in 1918 was the increase between 1918 and 1919?

7. In 1907, iron workers received 60¢ per hour. In 1916, they received 68¢ per hour. What per cent of their pay in 1907 was the increase?

8. In 1907, round steak sold for 14.3¢ per pound. In 1919, it sold for about 28.6¢ per pound. What per cent of the price in 1907 was the increase?

9. In 1907, flour sold for an average of 73.5¢ per sack of 25 pounds. In 1919, the average price was about $1.50 per sack. What per cent of the cost in 1907 was the increase?

10. In 1907, fresh eggs sold for an average price of 26.2¢ per dozen. In 1919, the average price was about 55¢ per dozen. What per cent of the price in 1907 was the increase?

11. In 1907, the average price for milk was 7.2¢ per quart. In 1919, it was about 15¢ per quart. What was the rate per cent of increase? (In this example, the question is asked in its more usual form. It means as before, what per cent of the price in 1907 was the increase?)

12. In 1907, the average price for potatoes was 21¢ per peck. In 1919, it was about 55¢ per peck. What was the rate per cent of increase?

13. *a.* Street car fares in Chicago on the surface lines were raised from 5¢ to 6¢ during the year 1919. What was the rate per cent of increase?

b. On the elevated lines the increase was from 5¢ to 8¢. What was the rate of increase?

14. Round steak sold in one town for 32¢ per pound in 1918. During the latter part of 1919, it sold for 24¢ per pound. What per cent of the price in 1918 was the decrease?

15. Farmers received $4.00 per hundred pounds for milk during the latter part of 1919, and $3.85 per hundred during the early part of 1920. What per cent of the price in 1919 was the decrease?

16. Bring to class any problems of increase or decrease that you can find, in which the rate is to be found.

EXERCISE 52

Saving by Wise Buying

1. A high grade of pork sausage sells for 45¢ per pound. A man found that he could buy fresh pork of the best quality, have it ground, and season it to suit himself at a cost of 30¢ per pound. What per cent of the higher price does he save by making his own sausage?

HINT. — B = "higher price"; P = the saving; R = ? See Notes § 38; also form of solution.

2. Evaporated milk is sold in 6-oz. cans for 10¢, or twelve cans for 92¢.

a. What would 12 cans cost if bought singly?

b. What per cent of this cost is saved by buying twelve cans at one time?

HINT. — B = "this cost"; P = the saving; R = ?

3. Free-running table salt can be bought for 10¢ per can, or three cans for 25¢. What per cent of the cost of three cans bought singly is saved by buying three cans at one time?

HINT. — B = "cost of three cans bought singly"; P = the saving; R = ?

4. A certain quality of rice can be bought for 18¢ per pound, or ten pounds for $1.65.

a. What would ten pounds cost if bought singly?

b. What per cent of this price is saved by buying ten pounds at one time?

5. Canned corn that sells for 20¢ per can is offered for sale at twelve cans for $2.25 or twenty-four cans for $4.35.

a. What per cent of the cost of twelve single cans is saved by buying twelve cans at one time?

b. What per cent of the cost of twenty-four single cans is saved by buying twenty-four cans at one time?

6. One package of macaroni can be bought for 15¢, or three packages for 42¢. What per cent of the cost of three single packages is saved by buying three packages at one time?

7. Gloves that were marked $4.00 sold for $3.50 during a sale. What per cent of the marked price was saved by buying the gloves during the sale?

8. Bed sheets marked $2.50 were sold for $2.25 during a sale. What per cent of the marked price was saved by buying the sheets during the sale?

9. Hats that had been marked $15.00 were sold during a sale for $5.00. What per cent of the former price was saved by buying such a hat during the sale?

10. Coffee, selling at 49¢ per pound, was offered for sale at three pounds for $1.35. What per cent of the cost of three single pounds was saved by buying three pounds at one time?

11. Vanilla extract is sold in a two-ounce bottle for 25¢; it is also sold in an eight-ounce bottle for 88¢.

a. What would eight ounces cost if bought in two-ounce bottles?

b. What per cent of this cost is saved by buying one eight-ounce bottle?

12. A family had been buying canned peas for 22¢ per can. They found that they could get twelve cans of equally satisfactory peas for $2.35. What per cent of the cost of twelve cans at 22¢ per can do they save by buying twelve cans for $2.35?

13. Rolled oats can be bought in bulk at the rate of ten pounds for 53¢. It is also sold in packages at the rate of two pounds for 28¢.

a. What do 10 pounds cost if bought in two-pound packages?

b. What per cent of this cost is saved by buying ten pounds in bulk?

14. A boy had been buying poultry feed for $4.25 per hundred pounds. He found that he could make a satisfactory feed by mixing 25 lb. of corn at 2½¢ per pound, 25 lb. of wheat at 5¢ per pound, 20 lb. of oats at 3¢ per pound, and 30 lb. of barley at 2½¢ per pound.

a. What did his own mixture cost him per hundred pounds?

b. What per cent of the greater cost did he save by mixing his own feed?

15. During the month of May, a family purchased 20 dozen of fresh eggs at 40¢ per dozen. They placed them in two stone jars. Over the eggs they poured a preserving mixture consisting of one part of water glass and eight parts of water. The water glass cost 50¢. They used the eggs during the following December, January, and February, when fresh eggs sold for an average price of 75¢ per dozen. All their eggs were as satisfactory as fresh eggs.

a. What did the eggs and water glass cost them?

b. What was the value of the eggs at the winter price?

c. What per cent of the value at winter prices did the family save?

16. Bring to class any problems of the kind studied in this list that you can find in your own town.

40. Application of finding the rate to problems of profit and loss.

Example 1. — A merchant bought a box of apples for \$3.00. He sold them for \$3.60. What rate per cent of the cost was the gross profit?

Solution. — 1. B = "the cost," \$3.00; P = "the gross profit," \$.60; R = ?
2. The formula is: $R = P \div B$.
3. Then the rate = $60¢ \div \$3.00 = \frac{1}{5} = 20\%$.

Remember that the merchant must pay all the expenses of conducting his business out of his gross profits. That which remains after all the expenses are paid is the net profit.

Example 2. — If the merchant in *Example* 1 estimates that it takes 10% of the selling price of his goods to pay his expenses, what per cent of the cost of the apples was his net profit?

Solution. — 1. The expenses = 10% of \$3.60 = $36¢$.
2. Then the net profit = $60¢ - 36¢ = 24¢$.
3. Then P = net profit = $24¢$; B = cost = \$3.00; R = ?
4. The formula is: $R = P \div B$.
5. Then the rate = $24¢ \div \$3.00 = .08 = 8\%$.

EXERCISE 53

1. *a.* What is meant by gross profit?

b. If the gross profit and the cost are known, how is the selling price found?

c. If the cost and the selling price are known, how is the gross profit, or the loss found?

2. In percentage problems, how do you pick out the number which is the base? How do you pick out the percentage?

3. What is the rule and the formula for determining the rate when the base and the percentage are known?

4. An article cost \$3.00 and sold for \$4.00.

a. What was the gross profit?

b. What per cent of the cost was the gross profit?

c. What per cent of the selling price was the gross profit?

5. An article that cost \$5.00 was sold for \$3.00.

a. What was the loss?

b. What per cent of the cost was the loss?

c. What per cent of the selling price was the loss?

6. In the following examples, what per cent of the cost was the gross profit or the loss?

Ex. No.	The Cost Was	The Selling Price Was	Ex. No.	The Cost Was	The Selling Price Was
6.	60¢	66¢	10.	90¢	81¢
7.	45¢	60¢	11.	\$2.50	\$3.00
8.	\$1.20	\$1.44	12.	2¢	4¢
9.	\$1.50	\$1.25	13.	3¢	9¢

14. A grocer bought eggs at 70¢ per dozen and sold them for 84¢ per dozen.

a. What was his gross profit per dozen?

b. What per cent of the cost was his gross profit?

c. If the cost of handling the eggs was 7¢ per dozen, what was the net profit per dozen?

d. What per cent of the cost was the net profit?

15. A grocer bought butter at 64¢ per pound and sold it for 72¢ per pound.

a. What per cent of the cost was his gross profit?

b. If the cost of handling the butter was 4¢ per pound, what was his net profit per pound?

c. What per cent of the cost was his net profit?

16. A window cost a dealer $4.20 and he sold it for $4.80. What per cent of the selling price was the gross profit?

17. If a suit of clothes cost the dealer $30.00 and it was sold for $40.00, what per cent of the cost was the gross profit? What per cent of the selling price was the gross profit?

18. If a shoe dealer makes a gross profit of $4.00 on a pair of shoes that he sells for $14.00, what did the shoes cost him? What per cent of the cost is his gross profit?

19. If a coal dealer makes a gross profit of $2.25 on a ton of coal that he sells for $8.50, what per cent of the cost is his gross profit?

20. A man receives a gross profit of $4.87½ on an article that cost him $75.00, and a gross profit of $3.63 on another that cost him $66.00. What per cent of the cost of each article is the gross profit on that article? On which article does he make the greater per cent of gross profit?

21. What per cent of the cost of peaches bought for $1.00 per box of 72 peaches is the gross profit, if the peaches are sold for 29¢ per dozen? Suppose that one dozen of the peaches are spoiled, and the balance are sold at 29¢ per dozen. What per cent of the cost is the gross profit in this case?

22. What per cent of the cost of apples bought for $3.00 per box of 96 apples is the gross profit if the apples are sold for 60¢ per dozen?

41. Finding the base when the percentage and the rate are known.

First notice that:

if some number $\times 5 = 300$, the number $= 300 \div 5 = 60$.
if some number $\times .04 = 240$, the number $= 240 \div .04 = 6000$.

This idea is used in solving:

Example 1. — Eight per cent of some number is 328. What is the number?

Solution. — 1. $R = 8\%$; $B =$ "some number" $= ?$; $P = 328$.
2. Then the base $\times .08 = 328$.
3. Then the base $= 328 \div .08 = 4100$.

Notice that the base equals the percentage divided by the rate.

Rule. — To find the base when the percentage and the rate are known, divide the percentage by the rate, after writing the latter either in decimal or common fraction form.

The formula is: $B = P \div R$.

Example 2. — 95 is 5% of what number?

Solution. — 1. $P = 95$; $R = 5\%$; $B = ?$
2. The formula is: $B = P \div R$.
3. Then the base $= 95 \div .05 = 1900$.

Example 3. — \$56 is 25% of what number?

Solution. — 1. $P =$ \$56.00; $R = 25\%$; $B = ?$.
2. The formula is: $B = P \div R$.
3. Then the base $= 56 \div \frac{1}{4}$.
$= 56 \times 4 =$ \$224.

EXERCISE 54

1. 30 is 2% of what number?
2. 35 is 5% of what number?
3. 10% of what number is 40?

4. 6% of what number is 420?
5. 48 is 8% of what number?
6. 7% of what number is 56.7?
7. 9% of what number is $18.27?
8. 162.4 is 4% of what number?
9. 275 is 5% of what number?
10. 15% of what number is 960?
11. $90 is 20% of what number?
12. 25% of what number is $2.50?
13. $13.25 is $33\frac{1}{3}$% of what number?
14. $16\frac{2}{3}$% of what number is 11.2?
15. $12\frac{1}{2}$% of what number is 6.5?
16. $234 is 125% of what number?
17. $60 is 120% of what number?
18. 210% of what number is 105?
19. 525 ft. is $33\frac{1}{3}$% of what number?
20. $113.75 is 35% of what number?
21. A man said that his taxes for a certain year were $125.10. It was known that the taxes of each man's property were 1.5% of the value of his property. What was the value of this man's property?

42. Finding the base when the result of adding to it or subtracting from it a certain per cent of itself is given.

Any quantity is 100% of itself.

Thus, 3 is 100% of 3; 19 is 100% of 19; etc.

If 5% of any number be added to the number, the result must be 105% of the number.

Check:

Thus 600 is 100% of 600.
30 is 5% of 600.
So, 630 is 105% of 600.

600
1.05
30 00
600 0
630.00

EXERCISE 55

How many per cent of a number is the result if

1. The number is increased by 10% of itself?
2. The number is increased by 18% of itself?
3. The number is decreased by 6% of itself?
4. The number is decreased by 25% of itself?
5. The number is increased by 20% of itself?
6. What number increased by 5% of itself is 315?

HINT. — 1. 315 = the number + 5% of the number. Then 315 must = 105% of the number. Therefore $B = ?$; $R = 105\%$; and $P = 315$.
2. What is the rule and formula for finding the base?
3. Complete the solution of the problem.

7. What number decreased by 10% of itself is \$630?

HINT. — \$630 = the number − 10% of the number. Then \$630 = 90% of the number. Complete the solution.

8. What number increased by 10% of itself is 478.5?
9. What number decreased by 25% of itself is 69?
10. What number increased by 5% of itself is \$176.40?
11. What number decreased by 8% of itself is \$230?
12. What number increased by 25% of itself is 150?
13. What number decreased by 25% of itself is 150?
14. What number decreased by 20% of itself is \$360?
15. What number increased by 20% of itself is \$360?

EXERCISE 56

Miscellaneous and Review Examples

1. What is meant by a rate per cent? What is the base? What is the percentage? How do you pick out the base? How do you pick out the percentage?

2. *a.* When the base and the percentage are known, how is the rate found? What is the formula?

b. When the rate and the base are known, how is the percentage found? What is the formula?

c. When the percentage and the rate are known, how is the base found? What is the formula?

3. Find the percentage when the base is \$378.45 and the rate $33\frac{1}{3}\%$.

4. Find the base when the rate is 6% and the percentage is \$426.24.

5. Find the rate when the base is \$372.40 and the percentage is \$409.64.

6. Write as a decimal: $\frac{1}{2}\%$; $3\frac{1}{2}\%$; 225%; 350%.

7. Find $3\frac{1}{2}\%$ of \$274.

8. Find 350% of \$274.

9. A dime is what per cent of \$2.00? of \$5.00? of \$10.00?

10. *a.* What is meant by increasing a number?

b. What is meant by decreasing a number?

c. How many per cent of a number is that number increased by 18% of itself?

d. How many per cent of a number is that number decreased by $7\frac{1}{2}\%$ of itself?

11. A man whose salary was \$5000 a year was given a bonus of \$1000 at Christmas time. How many per cent of his salary was this?

12. A man paid \$115.25 in taxes last year. He was told that taxes would increase about 6% this year. What would his taxes be, approximately?

13. A contractor estimated that the cost of building houses had increased about 60% in the past five years. What would it cost now to build a house that had cost $6300 five years before?

14. 40 rods is what per cent of a mile?

15. One square foot is what per cent of a square yard?

16. What number increased by 5% of itself is $971.25?

17. What number decreased by 18% of itself is $305.04?

18. A yard is what per cent of a foot?

19. A gallon is what per cent of a quart?

20. A bushel is what per cent of a peck?

21. *a.* A dry goods merchant advertised a discount of $33\frac{1}{3}\%$ on certain shopworn table linen. What would be the sale price of a tablecloth marked $6.45?

b. What is meant by a discount?

c. Tell two reasons why discounts are given by merchants.

22. Out of a salary of $2250 per year a man spends $112.50 for life insurance. What per cent of his salary is spent for life insurance?

23. Out of eight games played by a football team, the team won five. What per cent did it win and what per cent did it lose?

24. Another team won four games out of seven that it played. How does the per cent of games it won compare with the per cent won by the team in Example 23?

25. A man received $225 per acre for a farm that cost him $200 per acre. What per cent of the cost was his profit?

26. In one class, 14 pupils out of 28 were marked excellent; in another, 4 out of 24 were marked excellent. Find the per cent of each class marked excellent.

27. Find R when $P = \$917.00$ and $B = \$3275$.

28. Find P when $R = 62\frac{1}{2}\%$ and $B = \$74.64$.

29. Find B when $P = 435$ and $R = 15\%$.

30. A woman's hat cost \$5.00. It was marked for sale at a gross profit of 200% of the cost. At the close of the season it was sold at a discount of 50% of the marked price. What was the gross profit on the hat after the discount?

31. A suit that cost the dealer \$28.50 was marked to sell at an advance of 40% of the cost. At the end of the season, the suit was sold at a discount of 15% from the marked price. What was the final sale price? What was the gross profit on the suit if sold at the sale price?

32. If the pay a man receives now is more than that he received a year ago, how do you find what per cent of his former pay his increase is?

33. If you know the cost of an article and the selling price of it, tell how to find what per cent of the cost the gross profit is. Tell also how to find what per cent of the selling price the gross profit is.

34. If you know the selling price of an article, and know that the gross profit is a certain per cent of the selling price, how can you find the gross profit? How can you find the cost?

35. If a man knows how many tons of coal he burned, and how many pounds of ashes he has in his ash pile:

a. How can he find how many tons of ashes he has?

b. How can he find what per cent of his coal was ashes?

VII. INTEREST

43. Interest is money paid for the use of money. It is like rent paid for the use of a house, or farm, or automobile.

The amount of rent that one pays for a house or a farm depends upon the value of the property and upon the length of time that the property is rented. Similarly, the amount of interest paid for the use of money depends upon the amount of money borrowed and upon the length of time that it is borrowed.

Thus a man can borrow \$100 from a bank and keep it for a whole year by paying \$6 interest. If he keeps it for two years, he will have to pay 2 times \$6 or \$12; if he keeps it for only one half a year, he will pay one half of \$6 or \$3. If he borrows \$200 for one year, he will pay 2 times \$6 or \$12. For every \$100 that he borrows he will have to pay \$6 interest. Notice that he pays 6% of the amount he borrows, if he keeps it one year.

The money borrowed is called the **principal.** The interest for one year is a certain rate per cent of the principal; this rate is called the **rate of interest.** The year is regarded as consisting of twelve months of thirty days each, or of 360 days; this practice makes interest computation easier. The sum of the principal and interest is called the **amount.**

44. General method of computing interest.

Example 1. — A man borrows \$2000 for 3 years, agreeing to pay 6% interest annually. How much interest is

due at the end of 3 years? What is the amount due then?

Solution. — 1. The interest for 1 year = 6% of \$2000 = \$120.
2. The interest for 3 years = 3 × \$120 = \$360.
3. The amount in 3 years = \$2000 + \$360 = \$2360.

If the time had been one and one half years, the interest would have been $\frac{3}{2} \times \$120$ or \$180.

Rule. — To find the interest on any principal for a given length of time, at a given rate of interest, multiply the principal by the rate, and that result by the number of years in the time.

EXERCISE 57

Find the interest to be paid:

1. On \$500 borrowed for 1 year at 6% per year.
2. On \$500 borrowed for 2 years at 6% per year.
3. On \$500 borrowed for $\frac{1}{2}$ year at 6% per year.
4. On \$500 borrowed for $1\frac{1}{2}$ years at 6% per year.
5. On \$750 borrowed for 2 years at 5% per year.
6. On \$3000 borrowed for $\frac{1}{2}$ year at 8% per year.
7. On \$8300 borrowed for $2\frac{1}{2}$ years at 4% per year.

Find the interest and the amount to be paid:

8. On \$975 borrowed for $3\frac{1}{2}$ years at 5% per year.
9. On \$625 borrowed for $\frac{1}{4}$ year at 6% per year.
10. On \$1200 borrowed for $1\frac{1}{3}$ years at 8% per year.
11. On \$275 borrowed for 6 months at 5% per year.
12. On \$1500 borrowed for 4 months at 4% per year.
13. On \$1500 borrowed for 3 months at 5% per year.
14. On \$375.50 borrowed for 9 months at 6% per year.
15. On \$85 borrowed for 3 months at 8% per year.

16. Joseph Davison, having rented a farm, borrowed $1800 with which to buy tools, horses, and cows. He agreed to pay 5% interest. How much did he have to pay semi-annually (every six months)?

17. A certain bank pays to depositors interest on all money left on deposit for six months at the rate of 4% per year. A boy had saved $125. How much interest would he receive every half year if he deposited the money in the bank?

18. A farm boy borrowed from his father $50 with which to buy a calf. He agreed to pay interest on this sum every 6 months at the rate of 5% per year. How much money must he save each month to have his interest money ready at the end of six months?

19. John had heard his father talking about paying the interest "on the mortgage." He asked what his father meant. His father told him that he had had to borrow $4000 when he had built their home, and that he had given to the man from whom he had borrowed the money a paper called a mortgage, which would enable the lender to obtain possession of the property if John's father did not pay the interest or the principal when they became due. The interest was to be paid every half year at the rate of 6% per year. How much was the semi-annual interest?

20. What is the annual interest "on a mortgage" on a farm if the debt is $6500 and the rate of interest is 5%?

21. Make up one example like one of the last four in this list and solve it.

45. Finding the interest for any number of months.

Example. — What is the interest and the amount due *on $250 at the end of* 5 months if the rate is 6%?

Solution. — 1. The rate 6% means that 6% is charged for a whole year.

2. The interest for 1 year = .06 × $250 = $15.00.

3. Since 5 months is $\frac{5}{12}$ of a year,

the interest for 5 months = $\frac{5}{\cancel{12}} \times \cancel{\$15.00}^{\$1.25} = \$6.25$.

4. The amount = $250 + $6.25 = $256.25.

EXERCISE 58

Find the interest and the amount due on:

1. $575 borrowed for 2 months at 6%.
2. $800 borrowed for 7 months at 8%.
3. $650 borrowed for 1 month at 5%.
4. $375 borrowed for 5 months at 4%.
5. $495 borrowed for 11 months at 6%.
6. $250 borrowed for 1 year and 2 months at 8%.
7. $385 borrowed for 1 year and 7 months at 6%.
8. $150 borrowed for 1 month at 5%.
9. $425 borrowed for 11 months at 4%.
10. $1250 borrowed for 5 months at 7%.
11. Find the quarterly (3 months) interest at 6% on $2250.
12. Find the semi-annual (6 months) interest at 5% on $200.
13. Find the quarterly interest at 4% on $135.
14. Find the semi-annual interest at 5% on $625.
15. Find the quarterly interest at 8% on $375.

46. Finding the interest for any number of days.

Example 1. — What is the interest and the amount due on $250 borrowed for 17 days at 6%?

Solution. — 1. The interest on \$250 for one year = $.06 \times \$250 = \$15$.

2. 17 days = $\frac{17}{360}$ of a year so that the interest for 17 days = $\frac{17}{360} \times \$15.00$
$= \$\frac{17}{24} = \$.708 = \$.71$.

3. The amount = \$250 + \$.71 = \$250.71.

$$24\overline{)17.000}\quad .708$$
$$168$$
$$200$$
$$192$$

Example 2. — Find the interest on \$325 for 2 months and 13 days at 8%.

Solution. — 1. The interest for 1 year = $.08 \times \$325 = \$26.00$.

2. Then the interest for 2 months = $\frac{1}{6} \times \$26.00 = \$4.33$.

3. The interest for 13 days = $\frac{13}{360} \times \$26 = \frac{169}{180}$
$= .938$
$= \$.938$
$= \$.94$

4. Then the interest for 2 mo. and 13 da.
$= \$4.33 + \$.94 = \$5.27$

$$180\overline{)169.000}$$
$$1620$$
$$700$$
$$540$$
$$1600$$
$$1440$$

EXERCISE 59

Find the interest and the amount due on:

1. \$150 borrowed for 45 days at 6%.
2. \$275 borrowed for 20 days at 5%.
3. \$350 borrowed for 15 days at 8%.
4. \$425 borrowed for 40 days at 4%.
5. \$1500 borrowed for 21 days at 6%.
6. \$225 borrowed for 10 days at 5%.
7. \$175 borrowed for 18 days at 6%.
8. \$330 borrowed for 22 days at 6%.
9. \$4000 borrowed for 9 days at 5%.
10. \$3000 borrowed for 3 months 15 days at 6%.
11. *\$2500 borrowed* for 4 months 10 days at 5%.

12. $125 borrowed for 90 days at 6%
13. $215 borrowed for 75 days at 8%.
14. $3200 borrowed for 6 months 13 days at 6%.
15. $7500 borrowed for 9 months 22 days at 5%.
16. $3500 borrowed for 2 months 15 days at 6%.
17. $450 borrowed for 4 months 12 days at 5%.
18. $500 borrowed for 8 months 10 days at 7%.
19. $1500 borrowed for 5 months 18 days at 6%.
20. $825 borrowed for 3 years 5 months 19 days at 6%.

47. Finding the time between two dates.

Example. — 1. Find the time in years, months, and days between January 15, 1918, and May 8, 1920.

Solution. — 1. From January 15, 1918, to January 15, 1920, is 2 years.
2. From January 15, 1920, to April 15, 1920, is 3 months.
3. From April 15 to April 30 is 15 days.
4. From April 30 to May 8 is 8 days.
5. Adding, the total time is 2 years, 3 months, and 23 days.

NOTE. — Any pupil who does not know the number of days in each month, should memorize the following:

Thirty days has September,
April, June, and November,
All the rest have thirty-one,
Except the second month alone,
Which has twenty-eight
Except when leap year gives it twenty-nine.

The rule for determining when a year is a leap year is found on page 223.

Example 2. — Find the exact number of days from *January 14, 1920,* to June 22, 1920.

Solution. — 1. From January 14 to January 31 is 17 days
2. In February there are (Leap year) 29 days
3. In March there are . . . . . 31 days
4. In April there are . . . . . 30 days
5. In May there are . . . . . 31 days
6. From May 31 to June 22 is . . 22 days
7. Adding, the total time is . . . 160 days

EXERCISE 60

Find the time in years, months, and days:

1. From March 15, 1918, to August 21, 1920.
2. From February 13, 1919, to July 19, 1921.
3. From April 28, 1919, to December 11, 1920.
4. From May 14, 1918, to November 7, 1920.
5. From October 26, 1920, to December 17, 1921.
6. From May 22, 1919, to April 12, 1920.
7. From November 9, 1920, to June 23, 1921.
8. From February 17, 1920, to June 23, 1920.
9. From April 29, 1919, to January 13, 1921.
10. From December 12, 1920, to March 6, 1921.

Find the exact number of days:

11. From May 3, 1920, to November 24, 1920.
12. From April 26, 1921, to July 19, 1921.
13. From September 21, 1920, to December 12, 1920.
14. From January 7, 1921, to April 26, 1921.
15. From July 18, 1921, to January 11, 1922.

48. Finding the interest between dates.

Example. — Find the interest at 4½% on $500 borrowed *on November 16, 1920,* and repaid on December 21, 1920.

Solution. — 1. From November 16 to December 16 is 1 month.
2. From December 16 to December 21 is 5 days.
3. Then the total time is 1 month and 5 days.
4. The interest for 1 year $= .045 \times \$500 = \$22.50$
5. Then the interest for 1 month $= \frac{1}{12} \times \$22.50 = \$1.87$
6. And the interest for 5 days $= \frac{5}{360} \times \$22.50 = \$.31$
7. Adding, the interest for 1 month and 5 days $= \$2.18$

$$\frac{5}{360} \times 22.50 = \frac{112.50}{360} = .31$$

$$360\overline{)112.50}$$
$$108\ 0$$
$$4\ 50$$
$$3\ 60$$
$$90$$

EXERCISE 61

1. Find the interest at 6% and the amount due on $235 borrowed on May 13, 1920, and repaid on November 28, 1920.

2. Find the interest at 5% and the amount due on $650 borrowed on March 26, 1919, and which was repaid on May 28, 1920.

3. Find the interest at 7% and the amount due on October 15, 1920, on $1500 borrowed on July 24, 1920.

4. Find the interest at 4% and the amount due on April 23, 1921, on $2500 borrowed on June 11, 1920.

5. Find the interest at 8% and the amount due on August 9, 1920, on $875 borrowed on April 22, 1919.

6. When an owner of a Liberty Bond is compelled by circumstances to sell it, he should receive not only the market value of the bond but also the interest on the bond from the last interest payment date to the date on which he sells the bond.

How much interest is due on a $100 Liberty Bond from May 15, 1920, to July 1, 1920, if the rate of interest is 4%?

7. Find the interest on a $500 Liberty Bond on which the interest rate is $4\frac{1}{2}$% from September 15, 1920, to December 5, 1920.

8. Find the interest due on a Liberty Bond for \$50 from June 15, 1920, to July 30, 1920, if the rate is $4\frac{3}{4}\%$.

9. A man borrowed \$3500 from his bank on January 18, 1920, agreeing to pay $5\frac{1}{2}\%$ interest. What amount was due on February 16, 1920?

10. What amount was due on September 18, 1920, on \$1250 borrowed on May 23, 1919, if interest was charged at the rate of $5\frac{1}{2}\%$?

49. A promissory note is a written promise of one (or more) persons to pay a specified sum of money to another person.

\$375.00

Bayfield, Ill., March 11, 1920

Three months after date I promise to pay to the order of William Jamieson

Three hundred seventy five and $\frac{no}{100}$ *Dollars at* The First National Bank

with interest at 6 *per cent*

Due June 11, 1920 Charles Hendricks

The person promising to pay is called the **maker** of the note.

The person to whom payment is promised is called the **payee**. If the note reads " to the order of " some person, then that person may sell the note to some one else, who then becomes the payee. If the first payee sells the note, he must write his name across the back, at the left hand end of it. This is called **indorsing the note**. By so in-

dorsing the note, he becomes responsible for its payment, if the maker of the note fails to pay it when it falls due. If the payee writes only his name, then he indorses it "*in blank.*"

The amount of money specified in the note is called the **face** of the note. If interest is to be paid, then the words "with interest" at some rate per cent must appear in the note. Such a note is called an **interest-bearing note.**

The date when the note should be paid is called the **date of maturity.**

When the time of payment is specified the note is a **time note.**

The following is a **demand note.**

$400

Madison, Wis., Dec. 16, 1920

On demand I promise to pay to the order of

Henry B. Smith

Four Hundred and no/100 Dollars

at the First National Bank

with interest at 6 per cent

Charles R. Atkins

Such a note must be paid whenever the payee asks for payment.

If the time is given as a number of days, then the *actual number* of days is meant. If the date of maturity occurs on Sunday, or on a holiday, or on a Saturday half holiday, *the note* may be paid on the next business day.

EXERCISE 62

1. Write a note, dated to-day at your home town, in which you promise to pay James Hanson or order $425, with interest at 6%; make it payable at some bank in your town, six months from to-day.

2. In the note of Example 1, who is the payee? Who is the maker? What is the face of the note? What is the date of maturity?

3. What is the amount due on the note in Example 1 on the date of maturity?

4. Write a note for $2000 with interest at 5%, to be repaid one year from to-day, at some bank in your town. Let Charles Stanley be the maker and Edward Faucett or order, the payee.

5. What is the date of maturity of the note in Example 4? What is the amount due on this date?

6. Edward Adams, of Newberry, Conn., bought an automobile from the Peyton Auto Company on March 15, 1920. As part payment he gave them a note for $950, with interest at 6%, payable in 3 months at the office of the Peyton Auto Company.

a. Write the note.

b. Find the amount due on the note on the date of maturity.

7. George Erickson, of Athens, Penn., bought a farm on April 28, 1919, from Charles Anderson. As part payment he gave Mr. Anderson, or order, one note for $2000, with interest at 5%, payable at the Farmers Bank 6 months after date; and a second note, for $3000, with interest at 6%, payable at the same place 1 year and 9 *months after date.*

a. Write the two notes.

b. Find the amount due on the first note on the date of maturity.

c. Find the amount due on the second note on the date of maturity.

8. Eugene Turner of Middleton, Kansas, bought a corn planter on March 20, 1920, from the Atlas Hardware Company. As part payment he gave them a note for $125 with interest at 5% payable at the Exchange Bank nine months after date.

a. Write the note.

b. Find the amount due at maturity.

9. Henry Thompson borrowed $825 of the Exchange Bank on April 12, 1920. They had him sign a demand note, with interest at 7%.

a. Write the note.

b. If the bank asked for payment on April 29, 1920, what amount was due the bank?

10. Alfred Dewey borrowed from the Merchants Bank of Lima, New York, $1125, on September 6, 1920. He gave them a demand note with interest at $5\frac{1}{2}\%$.

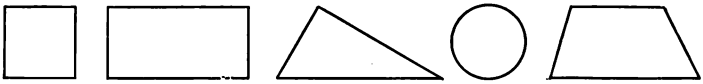
a. Write the note.

b. What amount was due on this note on November 16, 1920?

11. Make up one example like the ones in this list and solve it.

VIII. STRAIGHT LINES AND ANGLES

50. In geometry, figures like the **square**, the **rectangle**, the **triangle**, the **circle**, and the **trapezoid** are studied.



SQUARE RECTANGLE TRIANGLE CIRCLE TRAPEZOID

Some of these figures occur in objects that appear in nature, and all of them in objects that are made by human beings. Every one should know them, should know how to draw them, should know some of the important facts about them, and should know how to measure them. Geometry furnishes this information.

NOTE 1. — Material needed for the classroom work in geometry.

a. The pencil sharpener suggested at the beginning of the text.

b. One ball of common string.

c. A number of blackboard rulers. (Yardsticks are good.)

d. One or more pairs of scissors.

e. Some black or green tissue paper, to be used in making tracings of figures drawn on the board.

NOTE 2. — Material needed by the pupils for geometry work.

a. A supply of white unruled paper. All pupils should have the same kind of paper. Sheets about 6″ by 9″, that come in pads, or a notebook of about that size are good. For the best results in geometry instruction, all satisfactory work done by each pupil should be retained. If loose sheets are used, each pupil should have an envelope or a cardboard box in which to keep the completed work.

b. A No. 2 lead pencil with a good eraser on the end of it, or, *better still, a separate* eraser. The harder pencils may be preferred by *some teachers.*

c. A combination geometry tool, consisting of a straightedge, a right angle, a protractor, and an English-metric scale. One such card is furnished with this text.

(In this text, all the examples are stated in terms of English units of measure. If practice in the use of metric units is desired, the teacher can readily dictate dimensions involving such units for additional drill examples, similar to the ones given in the text.)

d. Some squares of transparent white paper, about 6″ by 6″. (Have the class purchase some sheets of white tissue paper, and cut it into squares of the desired size.)

e. A small quantity of squared paper, ruled into squares about one eighth of an inch on a side. (Can often be obtained at 10¢ stores. Not needed until later.)

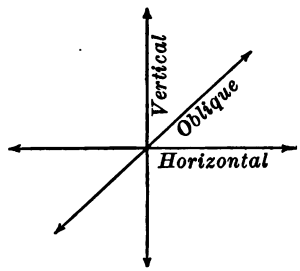
f. Later, or at once, if the teacher desires, compasses. (The lessons of the text are so arranged that compasses are not needed until the work on page 176 is studied, but so that any teacher who wishes to do so may introduce work with compasses earlier.)

51. Straight lines.

EXERCISE 63

1. Place on paper a small dot. Near it, print a capital letter *A*. This dot represents point *A*. Below it, print: **Point A.**

2. Place on paper a point *O*. Through it, draw a horizontal line, with arrowheads at its ends. Alongside it, print: **Horizontal line.**



The arrowheads mean that the line extends infinitely far in each of two directions. The line is infinitely long.

3. Through point *O*, of Example 2, draw a vertical line, terminated by arrowheads. Beside it, print: ***Vertical line.***

4. Through point O, of Example 2, draw an oblique line, terminated by arrowheads. Beside it, print: **Oblique line.**

5. Draw a second oblique line through point O.

6. Can you draw any more oblique lines through point O?

7. *a.* Fold a piece of paper, and carefully crease the fold. What kind of line is the creased fold? Test it with your straightedge.

b. Repeat this exercise with a second piece of paper. Do you obtain the same result?

8. Draw a straight line, terminated by arrowheads. Draw a second straight line crossing the first one. Print a capital M near the crossing point. Below, print or write: **Two intersecting lines.** Point M is the **point of intersection.**

(After you have completed this example, read again what you have written and make certain that you understand it.)

9. Draw a horizontal line. Below it, draw a second horizontal line.

a. Judging from appearances, will these two lines ever meet, even if they are extended infinitely far?

b. Can you find lines like these in your classroom?

c. Do you know what these lines are called?

d. Below this figure, print or write: **Two horizontal parallel lines.**

e. Memorize: *Parallel lines do not meet, even when extended infinitely far.*

10. Draw three vertical parallel lines. Print: **Three vertical parallel lines.**

11. Draw three oblique parallel lines. Can you find *any oblique parallel* lines in your classroom?

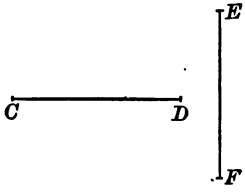
52. Line segments. Comparing; estimating; measuring.

EXERCISE 64

1. *a.* Place on paper two points A and B, a short distance apart. Draw a straight line from A to B. Do not place arrowheads at the ends, as you have been doing up to this point. Points A and B are the ends of segment AB. Below, print: **Segment AB.**

b. Memorize: *A segment, or a line segment, is the part of a straight line between two points of the line. It has two end points.*

2. Without measuring the segments CD and EF at the right, decide which is the longer, or whether they are of equal length. After you have decided, compare them by one or more of the following means:



a. On tracing paper, take a copy of CD and place this copy over EF. You can then tell which is the longer segment.

b. Along the straight edge of a piece of paper, take a segment equal to CD. Place one end of this segment on E and decide whether the other end falls on F. Again you can tell which is the longer segment, or whether they are equal.

c. If you have compasses, separate the points until one point can be placed on C and the other, at the same time, on D. Then, without changing the distance between the points of the compasses, place one point on E and decide whether the other can be made to fall on F. Again you can tell whether the segments are equal or not. This is the most accurate means of comparing two segments.

NOTE. — At the blackboard, all three means can be used. A piece of string may take the place of the compasses. Black or green tissue paper must be used for tracing paper.

The pupils should learn to use the first of the means proposed. If compasses are in their hands at this time, then the second means *suggested above* need not be used.

3. Draw on paper a horizontal segment MN. Beside it, without measuring, draw a vertical segment RS which you think is equal to MN. After you have done your best, compare the segments you have drawn by one or more of the means suggested in Example 2. Continue until you have a vertical segment that is equal to MN, approximately. Below, print: **Equal segments.**

4. *a.* Above segment MN of Example 3, draw, without measuring, an oblique segment that you think equals MN. Test it.

b. Draw a second oblique segment, in a different direction, which is also equal to MN. Test it.

5. Draw a segment XY. Then, using either compasses or the straight edge of a card as a means of transferring segment XY, draw a segment that equals: *a.* segment XY; *b.* twice segment XY; *c.* three times XY.

6. Draw a segment AB of reasonable length. Then, without measuring it, place a point C at what you think is the center of AB. After you have done your best, test the accuracy of your judgment by one of the means suggested in Example 2. If AC does not equal CB, erase point C, and locate it again and again until AC does (approximately) equal CB.

Below, print or write: **Point C bisects AB.** *C divides AB into two equal segments. C is the mid-point of AB.*

Now read carefully what you have written.

7. Draw a short segment CD. On a piece of paper, obtain a segment that equals CD. By folding the paper, find the mid-point of this segment. Then, placing the paper again beside CD, locate the mid-point of CD.

NOTE. — A segment can be bisected most accurately by means of a construction made by compasses. This construction is given in § 78, *p. 187. If desired, this* construction can be studied now, if the class *have compasses.*

8. Draw, without measuring, a segment MN about 4 inches long. Upon it, without measuring, locate three points which you think divide MN into four equal segments. Test the accuracy of your judgment. Continue until you have divided MN into four segments that are approximately equal. Below, print: MN divided into four equal segments.

9. In Example 6, what per cent of AB is AC? What per cent of BC is AB?

10. In Example 8, what per cent of MN is each of the parts of MN? What per cent of any one of the parts of MN is the whole segment MN?

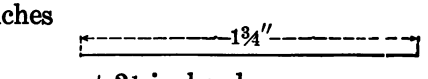
11. *a.* Draw a segment MN. On it, place points that you think divide it into three equal parts. Test the accuracy of your judgment. When you have finally separated MN into three parts that are (approximately) equal, print or write: MN separated into three equal parts.

b. What per cent of MN is each of the parts of it?

c. What per cent of any one of the parts is the whole segment MN?

NOTE. — If the pupils have compasses, they will be interested in the trial method of dividing a segment into equal parts. Thus, to divide a segment into three equal parts, take on the compasses a segment that appears to be one third of the given segment. Apply this to the segment three times, to test whether it is the proper length. If it is too long, or too short, make the proper change, and repeat until a segment one third as long as the given segment is obtained.

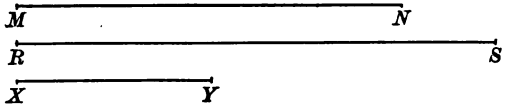
12. *a.* Draw and mark as in the adjoining figure a segment that is $2\frac{1}{4}$ inches long.



b. Draw and mark a segment $3\frac{1}{8}$ inches long.

c. Draw and mark a segment $1\frac{5}{8}$ inches long.

13. *a.* Without measuring, mark lightly above each of the segments below what you estimate its length to be, — giving the result in inches and eighths or quarters of an inch.



b. Now measure each of the segments carefully, and write the correct length beside the estimated length.

14. After the class have agreed upon the lengths in Example 13, answer the following questions:

a. What per cent of MN is RS?

b. What per cent of RS is XY?

c. What per cent of MN is XY?

d. What per cent of XY is MN?

15. *a.* Draw on the board a horizontal segment of reasonable length. (At least two feet.) Have each member of the class estimate its length, writing the result on a slip of paper. Then have five different pupils each measure the segment, giving the results in secret to the teacher. After any obviously inaccurate results have been thrown out, have the class find the average of the accepted measurements.

b. Repeat this exercise for three or four segments.

16. Estimate the length of some segment in the classroom, such as the length or width of a pane of glass, the width of the door, the width of the teacher's desk, the length of a page of this book. After estimating the *length, have the* segment measured by at least three *pupils.*

17. Are there in the classroom any parallel segments? Any oblique segments? Any horizontal segments? Any vertical segments?

18. *a.* Place on paper points A, B, and C, not all in one straight line. Draw segments, AB, BC, and AC.

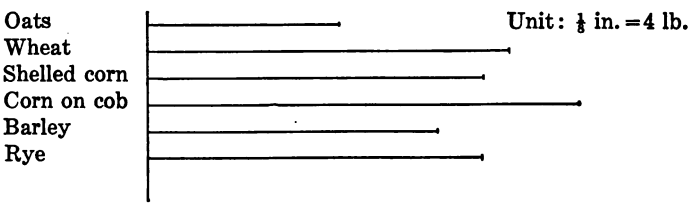
b. Measure AB, BC, and AC. Compare the sum of AB and BC with AC. Also compare the sum of AC and AB with BC. Also compare the sum of BC and AC with AB.

c. Complete the following sentence: *The straight line segment between any two points is the —— distance between the points.*

NOTE. — By this time the pupils should understand that measurements must be made with extreme care; that, even then the members of the class will get different results when measuring the same segment; that the most careful measurement gives only an approximate length.

53. Segments; Scale drawing; Line graphs.

The figure below is a scale drawing or line graph representing the weights of a bushel of the grains mentioned.



In this graph, $\frac{1}{8}$ inch on each segment represents 4 pounds. This is the **unit length.** The segment opposite "oats" is 1 inch long, as you will find if you measure it. Since 1 inch is 8 one-eighths of an inch, then a bushel *of oats must* weigh 8×4 lb. or 32 pounds.

EXERCISE 65

1. *a.* Measure the segment opposite the word "wheat." How much then must a bushel of wheat weigh?

b. Similarly, determine the weight of a bushel of each of the other grains.

2. *a.* What part of the weight of a bushel of corn on the cob is the weight of a bushel of shelled corn?

b. What part of the weight of a bushel of wheat is the weight of a bushel of oats? Of a bushel of rye? Of a bushel of barley?

c. What per cent of the weight of a bushel of barley is the weight of a bushel of oats? Of a bushel of wheat? Of a bushel of shelled corn? Of a bushel of corn on the cob?

3. *a.* Draw a horizontal segment AB, making it $\frac{1}{2}$ inch long. Above it, print 8 lb. Using this segment as your unit, draw a horizontal segment:

b. CD to represent 24 lb.; *e.* IJ to represent 10 lb.;

c. EF to represent 36 lb.; *f.* KL to represent 21 lb.;

d. HG to represent 14 lb.; *g.* MN to represent 42 lb.

4. *a.* Draw a vertical segment AB, making it 1 in. long. Beside it, print "16 feet."

Using this segment as your unit, draw a parallel segment:

b. CD to represent 20 ft.; *e.* IJ to represent 28 ft.;

c. EF to represent 54 ft.; *f.* KL to represent 56 ft.;

d. GH to represent 12 ft.; *g.* MN to represent 70 ft.

5. Represent by horizontal parallel segments the lengths of time that it takes seeds to germinate if, on the average:

bean germinates in 7 days;
beet germinates in 8½ days;
carrot germinates in 15 days;
corn germinates in 6½ days;
cucumber germinates in 8 days;
lettuce germinates in 7 days;
onion germinates in 8½ days;
pea germinates in 8 days;
radish germinates in 4½ days;
tomato germinates in 9 days.

(Let ¼ inch represent 1 day; beside each segment print the name of the seed.)

6. Represent by horizontal parallel segments the number of years that the following seeds live, if on the average:

beans live 3 years;
beet lives 6 years;
carrot lives 5 years;
cucumber lives 10 years;
onion lives 2 years;
pea lives 3 years;
radish lives 5 years;
tomato lives 4 years.

(Let ¼ inch represent 1 year; beside each segment print the name of the seed.)

7. Represent by vertical parallel segments the temperature readings on a day when:

at 8 A.M. the reading was 16°;
at 9 A.M. the reading was 18°;
at 10 A.M. the reading was 21°;
at 11 A.M. the reading was 24°;
at 12 M. the reading was 25°;
1 P.M. the reading was 26°;
2 P.M. the reading was 26°;
3 P.M. the reading was 25°;
4 P.M. the reading was 23°;
5 P.M. the reading was 21°.

(Let ¼ inch represent 1°. Below each segment print the hour.)

8. An arithmetic class kept a record of the class averages on daily drill work, for one week. Represent these averages by horizontal parallel segments.

Monday, 72%; Tuesday, 75%; Wednesday, 79%; Thursday, 73%; Friday, 80%. (Let ⅛ inch represent 2%.)

9. Represent by vertical segments the average heights of children of the following ages:

Age 1.....2 ft. 5 in.;	Age 7.....4 ft. 0 in.;
2.....3 ft. 0 in.;	8.....4 ft. 2 in.;
3.....3 ft. 4 in.;	9.....4 ft. 4 in.;
4.....3 ft. 6 in.;	10.....4 ft. 6 in.;
5.....3 ft. 8 in.;	11.....4 ft. 8 in.;
6.....3 ft. 10 in.;	12.....4 ft. 10 in.

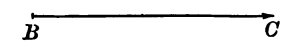
10. Represent by horizontal segments the average weights of children of the following ages:

Age 1.....weight 24 lb.;	Age 7.....weight 52 lb.;
2.....weight 32 lb.;	8.....weight 56 lb.;
3.....weight 36 lb.;	9.....weight 62 lb.;
4.....weight 41 lb.;	10.....weight 68 lb.;
5.....weight 45 lb.;	11.....weight 74 lb.;
6.....weight 49 lb.;	12.....weight 80 lb.

54. Angles. Kinds.

EXERCISE 66

1. Place on paper a point B. From B, draw to the right a straight line, terminated by an arrowhead. Letter the end point C. The arrowhead means that the line extends infinitely far in the direction indicated by the arrow. Below, print or write; A **half line** or **ray** BC.



Memorize: *A ray extends infinitely far in one direction from a point.*

2. We speak of a ray of sunlight. What is the "point" from which the ray starts?

3. How many end points does a segment have? A *line? A ray?*

4. On paper, place a point C. From C, draw five different rays. Print or write: *One ray can be drawn from a point in each direction.*

5. *a.* On paper, or on the board, place a point O. From it, draw ray OA; also, draw from it ray OB in a different direction. Below, print or write: **Angle AOB**, or $\angle$ **AOB**. Along AO, print **side**; likewise along OB. Near point O, print **vertex.**

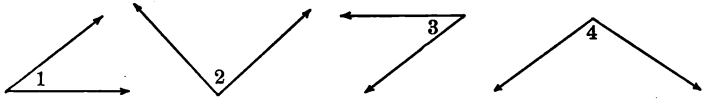
In the name of the angle, $\angle AOB$, the vertex letter O is between the two letters A and B.

Memorize: *An angle is the figure formed by two rays drawn from the same point.*

b. All the class have not drawn the same kind of angle, — probably. Have the different kinds now drawn on the board, but do not take the time to name them.

c. Since the sides of an angle are rays, they extend infinitely far.

6.



a. Make a copy of $\angle 1$ on tracing paper. Place this copy over $\angle 2$. Which is the larger angle?

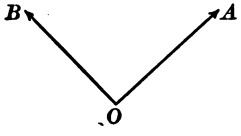
b. Similarly compare $\angle 1$ and $\angle 3$; also $\angle 1$ and $\angle 4$.

c. Compare $\angle 2$ and $\angle 3$; also $\angle 2$ and $\angle 4$.

d. Compare $\angle 3$ and $\angle 4$.

e. Memorize: *Two angles are equal if they can be made to coincide,* — that is, if one can be made to exactly fit over the other.

7. *a.* An angle like the one adjoining is a **right angle.** The edges of a card that meet at one corner usually form a right angle. The geometry card furnished with this text has such corners. Test the angle *AOB* adjoining by comparing with it the right angle at the corner of your card.



b. Using as pattern right angle the one at the corner of your card, draw three right angles, with the sides pointing in different directions.

Below, print: **Right angles.**

c. Do you see any right angles in your classroom? Test them if possible.

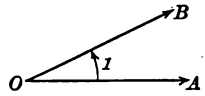
8. *a.* Fold a piece of paper in any direction and carefully crease the fold, thus obtaining a creased straight line.

b. Now fold the paper again, so that one part of the creased straight line folds over on the other part. Crease this fold also.

c. What kind of angle is formed by the creases in the paper? Test it.

9. *a.* An angle like the one adjoining is an **acute angle.**

b. Is this angle larger or smaller than a right angle? Test it.

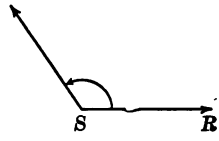


c. Draw on paper two acute angles. Below, print: **Acute angles.**

d. Complete and memorize: *An acute angle is an angle* —— *than a right angle.*

e. Are there any acute angles in your classroom? Test *them if possible.*

10. *a.* An angle like the one adjoining is an **obtuse angle.**



b. Is this angle larger or smaller than a right angle? Test it.

c. Draw on paper two obtuse angles. Below, print: **Obtuse angles.**

d. Complete and memorize: *An obtuse angle is an angle* ——— *than a right angle.*

e. Are there any obtuse angles in your classroom?

55. Right angles and perpendiculars.

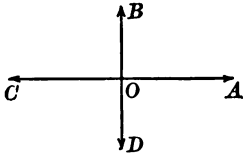
EXERCISE 67

1. *a.* Draw, without measuring, an angle which you think is a right angle. Then measure it. If accurate, print: Right angle; if inaccurate, print: Not a right angle.

b. Continue until you have drawn two angles that are (approximately) right angles.

2. *a.* Using the right angle at the corner of your card as the pattern, draw on paper a right angle. Letter it *AOB.*

b. Extend (make longer) the side *OA* in a straight line through the vertex *O* to a point *C.* Also extend the side *OB* through the vertex *O* to a point *D.*



c. What kind of angles are angles *BOC, COD,* and *DOA*? Test them. Print or write: Lines *AC* and *BD* intersect at right angles. They are **perpendicular.**

d. Memorize: *Two lines that form a right angle, or intersect at right angles,* are perpendicular lines.

3. Do you see any objects in the room that have edges or lines that are perpendicular? Test the angles formed by them.

4. *a.* Draw a horizontal line XY. On it place a point Z. Using your card, draw a perpendicular to XY at Z. To do this place one corner of the card at Z, with one edge along XY; then draw a line along the other edge that comes to point Z. Letter the end of this perpendicular W.

b. Repeat this exercise, starting with a vertical line.

c. Repeat this exercise, starting with an oblique line. (Give further drill on this construction, until all the pupils can do it.)

d. Print or write: ZW is perpendicular to XY at Z.

e. Does a perpendicular to a line necessarily go "straight up and down"? What is a line called that goes "straight up and down"?

NOTE. — The method of constructing a perpendicular to a line at a point of the line by means of compasses is taught in § 81, p. 190. It can be studied at this time if desired. However, each pupil should know how to draw the perpendicular quickly as directed in Example 4, because this construction corresponds exactly to that most commonly used by draughtsmen.

5. *a.* Draw a horizontal line and at three different points of it, draw perpendiculars to it. If these perpendiculars are extended, will they ever meet? What name is given to such lines?

b. Repeat this exercise starting with a vertical line.

c. Repeat this exercise starting with an oblique line.

d. Below; print or write and memorize: *Perpendiculars to the same line are* ———.

6. *a.* Draw a horizontal line AB. Above it place a point C, about one inch from line AB. Using your card, draw a straight line through C that will be perpendicular to AB. (Place your card with one edge passing through point C, and the perpendicular edge lying along line AB.)

b. Place a point D below the line, and draw through it a perpendicular to line AB.

Print or write: A perpendicular to AB from point C.

c. Repeat this exercise, starting with a vertical line AB.

d. Repeat this exercise, starting with an oblique line AB.

e. Give further drill on this construction until all the pupils can do it.

NOTE. — The method of constructing by compasses a perpendicular to a line from a point not on the line is taught in § 80, p. 189. It can be studied at this time if desired. However, each pupil should know how to draw the perpendicular quickly, as directed in Example 6, because it corresponds exactly to the method most commonly used by draughtsmen.

7. *a.* Have a pupil draw a horizontal line at the board and place a point on the board above the line. Have all the class estimate the distance from the point to the line. Have one pupil draw the perpendicular from the point to the line. Have two pupils measure this perpendicular. How many pupils came within one inch of the correct distance?

b. Repeat this exercise for two or three points at different distances from the line.

8. Repeat Example 7, starting with a vertical line.

9. Draw a horizontal line at the board. Have one pupil place on the board a point that he thinks is 15 inches from the line. Have the perpendicular drawn and *measured.* Repeat this exercise for a few other points.

10. *a.* On paper draw a horizontal line. Place, without measuring, a point which you think is 3 inches above the line. Now draw the perpendicular to the line from that point and measure it. Along your perpendicular line mark its length as you have determined it by measuring.

b. Repeat this exercise with a point $2\frac{1}{4}$ inches from the line.

11. Draw an oblique line. At one side of it place a point which is about two inches from the line. From this point draw with your card the perpendicular to the line, and then measure the perpendicular. Then draw two other lines from this point to your oblique line, and measure them. How do their lengths compare with the length of the perpendicular? Below, print or write: *The perpendicular segment is the —— segment from a point to a line.*

56. Dividing angles into equal parts.

EXERCISE 68

1. **Bisecting an angle.** — Draw an acute angle AOB. From its vertex O, draw a ray OC inside the angle, *which you think divides* the angle into two equal angles. After it is drawn, test the two parts with your tracing paper. If inaccurate, erase OC, and repeat the drawing, until you have succeeded in separating $\angle AOB$ into two angles that are (approximately) equal.

Below, print: OC bisects $\angle AOB$. OC is the bisector of $\angle AOB$.

Memorize: *The bisector of an angle divides it into two equal angles.*

2. *a.* Does every ray that is drawn through the vertex *of an angle bisect* the angle?

b. Upon your paper, draw an angle XYZ which is bisected by a ray YW. Below print: YW bisects $\angle XYZ$.

c. Also draw an angle RST, with a ray SK inside it which does not bisect $\angle RST$. Below print: SK does not bisect $\angle RST$.

3. *a.* Draw an obtuse angle $\angle XYZ$. From the vertex Y, draw a ray that bisects the angle. Test the two parts by means of your tracing paper.

b. Now bisect each of the halves in the same way.

c. Below, print or write: $\angle XYZ$ is divided into four equal angles.

NOTE. — The method of constructing the bisector of an angle by compasses is taught in § 79, page 188. It can be studied at this time if desired.

4. *a.* Draw a large acute angle RST.

b. Without measuring, draw through the vertex two rays that you think divide $\angle RST$ into three equal angles. Test them by means of your tracing paper.

c. When you have finally divided $\angle RST$ into three parts that are approximately equal, print or write: $\angle RST$ divided into three equal angles.

57. Measuring angles. — The unit for measuring an angle is one **angle-degree**, or a **degree**. **A degree is one-ninetieth of a right angle.** Remember, that an angle-degree is a very small angle.

EXERCISE 69

1. How many degrees are there in one half a right angle?

2. How many degrees are there in one third a right angle?

3. How many degrees are there in one fourth a right angle?

4. How many degrees are there in one fifth a right angle?

5. How many degrees are there in 30% of a right angle?

6. How many degrees are there in $33\frac{1}{3}\%$ of a right angle?

7. How many degrees are there in $16\frac{2}{3}\%$ of a right angle?

8. What per cent of a right angle is 9°?

9. What per cent of a right angle is 50°?

10. What per cent of a right angle is 35°?

11. How many degrees are there in the angle formed by the hands of a clock at three o'clock?

12. How many degrees are there in the angle formed by the hands of a clock at two o'clock? At one o'clock?

13. How many degrees are there in the angle formed by the hands of a clock at four o'clock?

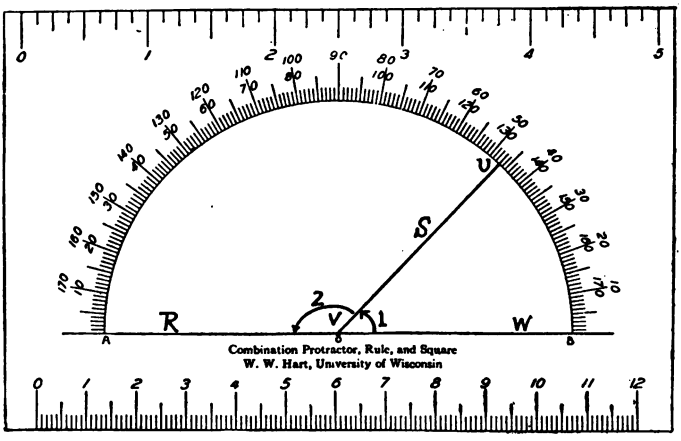
14. How many degrees are there in the angle formed by the hands of a clock at five o'clock?

15. Through how many degrees has the minute hand revolved in one hour? In 75% of an hour? In 60% of an hour?

58. **The protractor** is the tool for measuring an angle. A convenient form of protractor is part of the geometry card furnished with this text. On your card, find the center O of the protractor and the diameter AOB.

NOTE. — Observe that there are two scales of angular measures, — one starting at the point A and the other starting at the point B. *When AO is one side of* the angle, then the inner scale is used; when *BO is one side of the angle,* then the outer scale is used.

a. **Using the protractor to measure an angle.** — Place the card so that the center *O* of the protractor is on the vertex *V* of the angle, the diameter *AOB* is on the side *VW*, and the other side of the angle lies in the semicircular cutout of the card. (See the figure.) *SV* then cuts the protractor scale, and the size of the angle can be read. In the figure, ∠ 1 contains 47° and ∠ 2 contains 133°. Sometimes it may be necessary to extend the side *VS* so that it will cut the protractor scale.



b. **Using the protractor to draw an angle of given size.** —To draw an angle of 47° at point *V* of line *RW* and lying above *RW*:

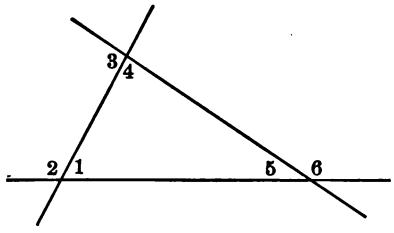
1. Place the diameter *AB* along *RW* with center *O* on point *V*, and the semicircular cutout lying above the line *RW*.

2. Starting from the side *VW*, find 47° on the scale, using the outer protractor scale. Opposite 47°, place a small point on the paper. Mark it point *U*.

3. Draw *VU*. Then ∠ WVU contains 47°.

EXERCISE 70

1. Draw on the board three intersecting lines as in the adjoining figure. Number the angles. Have two pupils measure each of the angles.



2. On the board, draw a horizontal line, with a point *O* on it.

a. Have one pupil draw a ray above the line, from point *O*, making an angle of 50° with the line. Print 50° inside the angle. Have a second pupil measure the angle to test the accuracy of the construction.

b. Have a third pupil draw a ray below the line from the point *O*, making an angle of 65° with the line. Have a fourth pupil test the accuracy of the construction.

c. Have another pupil draw from the point *O*, above the line, a ray that makes with the horizontal line an angle of 115°. Have it tested.

d. Do a few other exercises like these.

3. Draw on your paper a vertical line and place a point *P* on it. From *P*, draw a ray to the left of the line that makes an angle of 40° with the line, and also one to the right that make an angle of 100° with the line. Inside each of these angles, near its vertex, print its size.

4. Draw an oblique line, and place a point *M* on it. At point *M*, draw a ray that makes an angle of 80 degrees with your oblique line. Print the size of the angle, inside the angle near the vertex.

5. Have one pupil draw on the board an acute angle. Have all the rest of the class silently estimate the size of the angle. Then have two pupils measure the angle.

(Repeat this exercise for three or four angles, including obtuse angles.)

6. Draw on your paper an angle. Then, using your protractor, draw its bisector. Test the accuracy of your drawing by means of your tracing paper. Inside each of the halves, print its size.

7. Draw on paper an obtuse angle. Then without measuring it, draw a ray which you think bisects the angle. Then test the accuracy of your judgment by measuring each of the two angles formed. Print inside each the size of the angle.

8. Find in the room, if possible, two lines that form either an acute or an obtuse angle. Estimate the size of the angle. Then measure the angle.

9. If you can see, from your schoolroom window, a gable roof, measure with your protractor the angle made by the two sides of the roof at the gable. While your result will not be accurate, you will find that many of your classmates will obtain just about the same result that you do.

10. Bring to class any pictures or drawings that you can obtain that have on them lines that form angles. Estimate and measure some of the angles.

EXERCISE 71

Review of Lines, Segments, and Angles

1. Draw a horizontal line; a vertical line; an oblique line.

2. *Fold* a straight line; also fold a right angle.

3. Draw two intersecting straight lines; two parallel lines; two perpendicular lines.

4. Draw and properly mark a segment; a line; a ray.

5. Draw two equal segments.

6. Draw a segment and bisect it by sight.

7. If a segment $\frac{1}{4}$ inch long represents 10 tons, draw segments that represent 40 tons, 25 tons, and 65 tons respectively.

8. Draw a right angle; an acute angle; an obtuse angle.

9. Draw by sight two equal angles.

10. Using your drawing card, draw a perpendicular to a line at a point of the line; also a perpendicular to the line from a point not on the line.

11. Draw an angle, and then draw by sight the bisector of the angle.

12. With your protractor, draw an angle of 35° and also one of 125°.

13. What is a segment?

14. What is a ray?

15. What are intersecting straight lines?

16. What are parallel straight lines?

17. What are perpendicular straight lines?

18. When are two segments equal?

19. When is a segment bisected? What is the point called that bisects a segment?

20. What is an angle?

21. When are two angles equal?

22. When is an angle bisected? What is the ray called *that bisects* an angle?

23. How can you test two angles to determine whether they are equal?

24. What is a " straight up and down " line called?

25. What is one degree of angle measure?

26. How many degrees are there in a right angle?

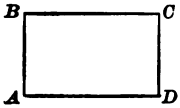
27. Tell some differences between a segment, a ray, and a straight line.

28. Tell two ways of testing two segments to determine whether they are equal?

29. How many rays can be drawn from a point in any one direction? How many can be drawn altogether from one point?

30. Does it change the size of an angle to extend its sides?

IX. RECTANGLES AND SQUARES



59. Rectangle. — The figure at the left is a rectangle.

NOTE. — In drawing classes, the rectangle is often called an *oblong*, but mathematicians call it a rectangle.

EXERCISE 72

1. By means of your tracing paper and other geometry tools, answer the following questions:

a. What kind of angles has a rectangle? Test them.

b. Compare the sides AD and BC, as to length. Test them.

c. Compare the sides AB and CD. Test them.

d. Since $\angle A$ is a —— angle, what kind of lines are AD and AB? What kind of lines are BC and AB?

e. What kind of lines are AD and BC? AB and CD? Why? (See Ex. 5, p. 126.)

f. Write out the following sentence, supplying missing words:

A rectangle has four sides and —— angles; its angles are —— angles; its opposite sides are —— and ——; the two sides meeting at one vertex are ——.

2. *a.* Draw a segment AB, 3 inches long.

b. At A, using your card to give you a right angle, *draw AD* perpendicular to AB, making it 2 inches long.

c. At *B*, draw *BC* perpendicular to *AB*, making it 2 inches long.

d. Connect points *D* and *C*.

e. Test the angles *A*, *B*, *C*, and *D*. Also compare the sides *AB* and *DC* by one of the means suggested in Ex. 2, p. 115.

f. What kind of figure have you drawn?

g. Draw *AC* and also *BD*.

h. Below print: **Rectangle** *ABCD*. Along *AB*, print **base**; along *AD*, **altitude**; along *AC*, **diagonal**.

3. *a.* Draw a rectangle whose base is 4 inches and whose altitude is 2.5 inches.

b. Draw and measure its two diagonals. On each diagonal, print its length.

c. Compare the diagonals also by other means. (See Ex. 2, p. 115.) Compare also the diagonals of the rectangle drawn for Example 2.

d. Below write: *The diagonals of a rectangle* —— ——.

e. Compare the two parts of one of the diagonals; also the two parts of the other diagonal. Do the same in the figure for Example 2.

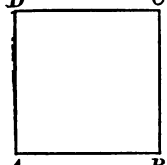
f. Below write: *Each diagonal of a rectangle divides the other into* —— —— *parts.*

4. *a.* In the figure for Example 2, determine whether the two angles into which the diagonal divides one of the angles of the rectangle are equal. (Use both tracing paper and your protractor.)

b. Similarly study the angles formed at each of the vertices of the rectangle, both in Example 2 and in *Example 3.*

c. Below, write: *A diagonal of a rectangle* (does or does not) *divide the angles through which it is drawn into two* —— *parts.*

5. Do you find in the classroom any rectangles? Any diagonals of a rectangle? Can you discover any instances of the facts you have studied in Examples 2, 3, and 4?



60. Square.—The figure at the left is a square.

EXERCISE 73

1. *a.* What kind of angles has a square?

b. Study the sides of a square and tell what you observe about them.

c. Write down what you have decided is true about the angles and the sides of a square.

2. *a.* Draw a square whose sides are 3 inches long. Make certain that the angles are right angles and that the sides are equal.

b. Draw the two diagonals and measure them. On each, print its length. Compare them in other ways. What seems to be true?

c. Repeat the exercise for a square whose sides are 2 inches long.

d. Below, write: *The diagonals of a square are* ——.

3. *a.* In the figures of Example 2, compare the two parts of one of the diagonals. What seems to be true? Repeat for the other diagonal.

b. Below, write: Each diagonal of a square —— *the other into* —— —— *parts.*

4. *a.* In the figures for Example 2, what kind of angles are formed by the two diagonals of a square at their point of intersection? Test them.

b. Below, write: *The two diagonals of a square form* —— *angles. The diagonals of a square are* —— *lines.*

5. *a.* In the figures for Example 2, observe the two angles into which one of the diagonals divides one of the right angles through which it is drawn. What seems to be true about them? Test them by means of tracing paper, and by means of your protractor.

b. Repeat this exercise at each of the vertices of a square.

c. Write: *A diagonal of a square* (does or does not) *divide the angles through which it is drawn into* —— —— *parts.*

6. *a.* Do the diagonals of a rectangle bisect the angles of the rectangle? Do the diagonals of a square?

b. Do the diagonals of a rectangle bisect each other? Do the diagonals of a square?

c. Are the diagonals of a rectangle perpendicular to each other? Are the diagonals of a square?

d. Are the sides of a rectangle all equal? Are the sides of a square?

7. How large is each of the small angles formed at the vertex of a square when a diagonal is drawn through that vertex?

8. Can you draw a four-sided figure whose sides are all equal, which is not a square?

9. Draw a rectangle *ABCD* whose base is 3.5 inches and whose altitude is 2.25 inches. Find *E* the mid-point of *AB*; *F* the mid-point of *BC*; *G* the mid-point of *CD*; and *H* the mid-point of *AD*. Draw *FG*, *GH*, *HE*, and

EF. Study the figure *EFGH*, and write what you observe about it. Is *EFGH* a square?

10. Repeat Example 9, starting, however, with a square whose side is 2.75 inches. What kind of figure is *EFGH* in this exercise?

61. Drawing rectangles and squares to scale.

EXERCISE 74

1. If a segment 1 in. long represents 15 ft., how long must a segment be to represent:

a. 30 ft.? *b*. $7\frac{1}{2}$ ft.? *c*. 45 ft.? *d*. $22\frac{1}{2}$ ft.?

e. What length is represented by a segment that is 4 in. long?

2. If a segment 1 in. long represents 20 ft., how long must a segment be to represent:

a. 30 ft.? *b*. 5 ft.? *c*. 40 ft.? *d*. $17\frac{1}{2}$ ft.? *e*. $32\frac{1}{2}$ ft?

f. What length is represented by a segment that is $3\frac{1}{4}$ in. long?

3. If a segment $\frac{1}{2}$ in. long represents 8 ft., how long must a segment be that represents:

a. 12 ft.? *b*. 20 ft.? *c*. 6 ft.? *d*. 2 ft.? *e*. 15 ft.?

f. What length is represented by a segment that is $2\frac{1}{2}$ in. long?

4. *a*. Draw to scale a rectangle 20 ft. by 85 ft., letting 1 in. represent 10 ft. (Use your geometry card to aid you in drawing good right angles at the vertices, and measure the sides very carefully.)

b. *Draw a diagonal* of this rectangle. Measure it as *accurately as you can* to sixteenths of an inch. Then

determine what length is represented by this diagonal, since 1 in. represents 10 ft.

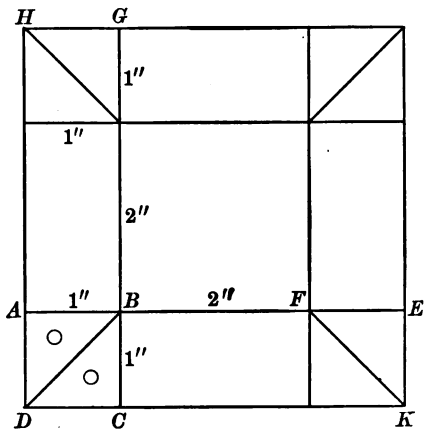
5. *a.* A baseball "diamond" is a square 90 ft. on a side. Draw a plan for such a diamond, letting 1 in. represent 20 ft.

b. Draw the diagonal representing the shortest distance from the home plate to second base. Measure this diagonal, and then determine the distance.

c. What is the shortest distance from first base to third base?

6. Draw the plan for a football field. Length 300 ft.; width 160 ft.; "five yard lines," parallel to the ends and five yd. apart; goal posts, $5\frac{1}{3}$ yd. apart; and at equal distances from the center of the goal lines. Let 1 in. represent 20 yd.

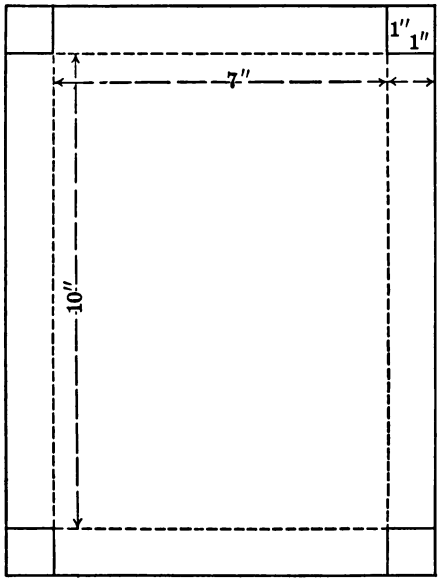
7. At the right is a pattern for a receptacle to hold salted nuts or candies for the individual guests at a party.



Rectangle *CGHD* is folded up on line *CG*, and rectangle *EKDA* on line *AE*. Then *CB* and *AB* are brought together and the corner is creased on line *DB*. Holes are then punched through as shown in square *ABCD*, and the corner is tied with small ribbon. Similarly, the other corners are folded and fastened.

Make the pattern for such a receptacle full size, cut it *from paper*, and fold it as directed.

8. At the left is a pattern for the bottom of a cardboard box to hold the papers and tools of a pupil in a geometry class. It is made to hold papers 6 in. by 9 in. in size.



a. Make a half-size scale drawing of this pattern or of one that will hold the kind of paper you are using, letting $\frac{1}{2}$ in. represent 1 in.

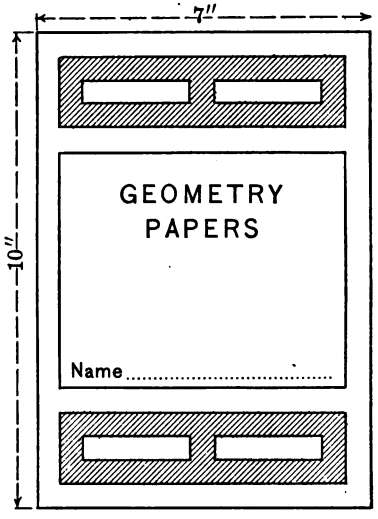
b. Also make the scale drawing of the pattern for the top of this box, if the top is $\frac{1}{8}$ in. longer and $\frac{1}{8}$ in. wider than the bottom, and the sides and ends are $\frac{3}{4}$ in. wide.

c. How large must a sheet of cardboard be in order to make the top of your box?

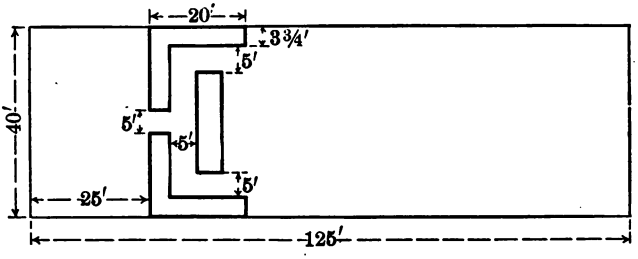
NOTE. — If you decide to make such a box, make your drawing on the side of your cardboard that will be the *inside* of your box. Cut out the square corners. Then with the point of a sharp penknife, cut *part way* through the cardboard on the dotted lines. Then turn the cardboard over, and bend up the sides and ends on the lines made by the knife. These will show through the cardboard. Then bind the corners together with gummed paper or gummed labels.

9. *At the top of facing page is a design for a decorated top for the box constructed in Example 8. Make a*

half-size drawing of such a design for your box. Do not necessarily copy this design.



10. Below is a plan for flower beds and a vegetable garden on a city lot. The dimensions are all given in the plan. Make a scale drawing of this plan, or make up a plan of your own for a lot of any size that you may select. This plan can be drawn on small paper by letting 1 in. represent 20 ft.

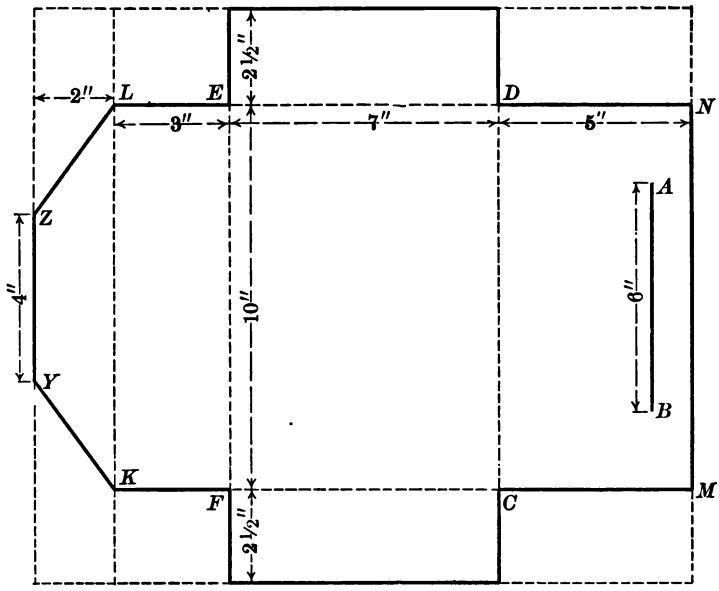


11. On page 144 is a pattern for an envelope in which to keep geometry papers, 6 in. by 9 in. in size. Ones like these were made by the pupils in certain schools.

a. Why is rectangle *CDEF* made 7 in. by 10 in.?

b. How wide and how long must the paper be to make this envelope?

c. *XYZW* is a flap to slip inside slit AB. How far *from the edge MN* must the slit AB be cut?

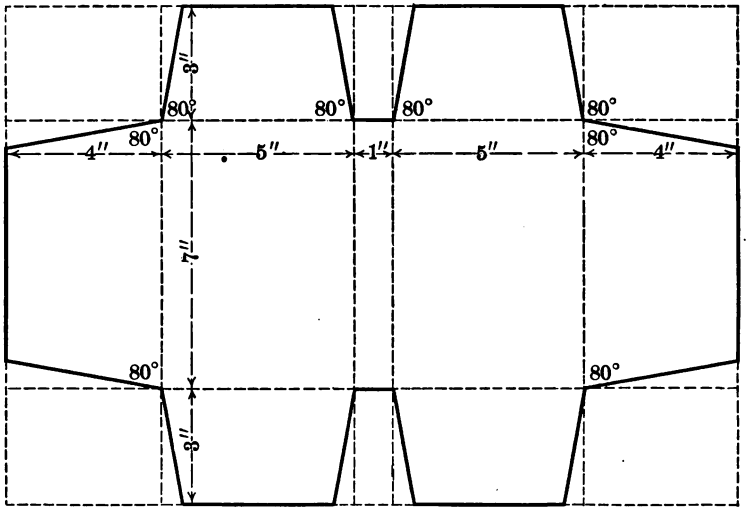


d. Make a scale drawing of this pattern, letting $\frac{1}{4}$ in. represent 1 in. (Or make a pattern for your own size of paper.)

NOTE. — *a.* Napkin cases and cases for tablecloths similar in design to this pattern are made from art linen or other suitable material. In such cases, the flap *XYZW* and slit *AB* are omitted. Corners *K* and *L* are fastened to the rectangle *CDNM* by snap fasteners.

b. You may be able to obtain sufficiently heavy paper for such envelopes at a hardware store. To fold on a line *CD*, place your ruler over rectangle *CDEF*, with one edge held fast on line *CD*. Then bend rectangle *MNDC* up and press the paper against the edge of the ruler.

12. On page 145 is a pattern for a book cover, to be *made from* heavy wrapping paper. This one is designed *for a book 5 in.* by 7 in. by 1 in.

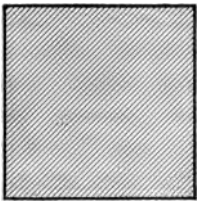


Measure your mathematics book, and make a quarter-size scale drawing of the pattern for such a cover for the book.

NOTE. — After you are certain that your pattern is satisfactory, why not obtain suitable paper, or cloth, and make a full size cover for your book?

Measuring Rectangles and Squares

62. Units of surface measure.



A square whose sides measure one inch **incloses a square inch** of plane surface. *The square itself is the boundary of the square inch.* Similarly a square one foot on each side incloses a square foot; one a yard on each side incloses a square yard; etc.

All these are **units of surface measure.** Review at this *time the table* of surface measure given on page 220.

EXERCISE 75

1. Draw a rectangle whose base is 4 inches and whose altitude is 3 inches. Draw the necessary lines to divide it into square inches. Shade one of these square inches. How many square inches are there within the rectangle?

2. Tell how many and what kind of square units there are within a rectangle:

a. If the base is 6 inches and the altitude is 5 inches;

b. If the base is 8 feet and the altitude is 6 feet;

c. If the base is 9 yards and the altitude is 7 yards;

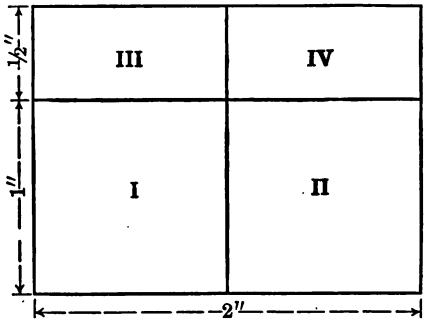
d. If the base is 11 rods and the altitude is 3 rods;

e. If the base is 2 miles and the altitude is 9 miles.

3. Draw a rectangle whose base is 1 inch and whose altitude is $\frac{1}{2}$ inch. What part of a square inch does it inclose?

4. Draw a rectangle whose base is 1 inch which incloses $\frac{3}{4}$ of a square inch.

5. At the right is a rectangle with base 2 inches and altitude $1\frac{1}{2}$ inches. Lines divide its interior into four parts. I and II are square inches; III and IV are halves of square inches. All together there are within the rectangle 3 square inches of surface. Notice that the number of square inches within the interior is $2 \times 1\frac{1}{2}$ or 3.



6. Make a drawing like that in Example 5 for a rectangle with base 4 inches and altitude $2\frac{1}{2}$ inches. How many square inches are there in its interior? How does this number compare with $4 \times 2\frac{1}{2}$?

7. Make a drawing like that in Example 5 for a rectangle with base $2\frac{1}{2}$ inches and altitude $1\frac{1}{2}$ inches. How many square inches are there in its interior? How does this number compare with $2\frac{1}{2} \times 1\frac{1}{2}$?

From the examples in Exercise 75, it is clear that: *to find the number of square units inclosed by a rectangle, multiply the number of linear units in the base by the number of the same units in the altitude.*

Rule. — The area of a rectangle is the product of its base and altitude.

If A represents the area, a the length of the altitude, and b the length of the base (measured by the same linear unit), then the formula for the area is

$$A = a \times b \quad \text{or} \quad A = ab.$$

This formula is used as follows:

Example. — Find the area of a rectangle whose base is 10 ft. 6 in., and whose altitude is 4 ft. 8 in.

Solution. — 1. $b = 10$ ft. 6 in. $= 10\frac{1}{2}$ ft.
$a = 4$ ft. 8 in. $= 4\frac{2}{3}$ ft.

2. Since $A = a \times b$,
then $A = 10\frac{1}{2} \times 4\frac{2}{3}$
$= \frac{21}{2} \times \frac{14}{3} = 49$ sq. ft.

EXERCISE 76

Find as in the example above the area of a rectangle when:

1. $a = 25$ *ft.*, and $b = 37$ ft.

2. $a = 40$ ft., and $b = 108$ ft.

3. $a = 16\frac{1}{2}$ ft., and $b = 20$ ft.

4. $a = 7\frac{1}{2}$ in., and $b = 26\frac{2}{3}$ in.

5. $a = 10\frac{1}{3}$ yd., and $b = 36$ yd.

6. $a = 11.5$ rd., and $b = 17.3$ rd.

7. $a = 16.25$ yd., and $b = 32.5$ yd.

8. $b = 14.8$ ft., and $a = 5.2$ ft.

9. $a = 18\frac{3}{4}$ ft., and $b = 35\frac{1}{6}$ ft.

10. $a = 5\frac{1}{2}$ in., and $b = 8\frac{3}{4}$ in.

11. How many tiles one foot square are needed to cover a porch floor that is 11 ft. wide and 18 ft. long?

12. How many square feet of roofing paper are there in a roll 36 ft. long and 36 in. wide?

13. How many square feet of wall paper are there in a roll of paper that is 18 in. wide and 16 yd. long?

14. A certain kind of rug costs $13.50 per square yard. What will a rug of that kind 11 ft. wide and 17 ft. long cost?

15. A contractor agreed to build a cement floor for a piazza for 30¢ per square foot. If the piazza is $24\frac{1}{2}$ ft. long and $9\frac{1}{2}$ ft. wide, what will the cost be?

16. What is the number of acres in a field 15 rd. long and 27 rd. wide?

17. What will be the cost of a flexo-tile floor for a bathroom which is 8 ft. wide and $11\frac{1}{2}$ ft. long, at 57¢ per square foot?

18. A man paid $3600 for a lot 40 ft. wide and 120 ft. long. How much did he pay per square foot?

19. A corner lot is 40 ft. wide and 150 ft. long. It is *proposed to build* a concrete sidewalk in front of it and on *the one side of it.* The walk is to be 5 ft. wide.

a. Make a scale drawing of the lot and the sidewalk, letting 1 in. = 20 ft.

b. Determine the cost of the sidewalk at 23¢ per square foot.

20. A sun parlor is 10 ft. 8 in. wide and 20 ft. 6 in. long. Determine the cost of a tile floor for it at 75¢ per square ft.

21. *a.* How many square inches of surface are there in the exposed surface of a brick that measures 8 in. by $2\frac{1}{2}$ in. on the exposed face?

b. How many such bricks, approximately, will be needed for a porch that is 22 ft. long and 9 ft. wide?

22. Determine the cost at 25¢ per square yd. of painting the roof of a garage if there are two surfaces, each $23\frac{1}{2}$ ft. by $12\frac{1}{2}$ ft.

23. One gallon of paint will cover 250 square feet of surface, two coats. How many gallons of paint will be needed to give two coats of paint to a barn 25 ft. wide, 60 ft. long, and averaging 21 ft. in height?

24. A kitchen is 11 ft. wide, 15 ft. long, and 9 ft. high. What will it cost to replaster the ceiling of this kitchen at 33¢ per square yard? The side walls, not making any allowance for doors or windows?

25. A living room is 14 ft. wide, $22\frac{1}{2}$ ft. long, and $8\frac{1}{2}$ ft. high. Find the cost of plastering the walls and ceiling at 47¢ per square yard, not making any allowance for doors or windows.

26. Linoleum comes in strips of any length 6 ft. wide or 12 ft. wide.

a. What will 8 ft. in length of this linoleum, 6 ft. wide, *cost at $1.95* per square yard?

b. What will $11\frac{1}{2}$ ft. in length, 12 ft. wide, cost at \$1.95 per square yard?

27. A room is 13 ft. wide and 18 ft. long. It is proposed to carpet it with carpet that is 27 inches wide, the strips running the long way of the room.

a. Make a scale drawing of this room showing the strips of carpet. Let $\frac{1}{4}$ in. represent 1 ft.

b. How many strips of carpet are required? How long is each strip?

c. What will this amount of carpet cost at \$3.25 per yard? (This means that each yard, 27 in. wide, costs \$3.25.)

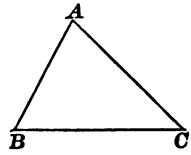
28. How many yards of carpet are needed for a room 20 ft. 6 in. long and 16 ft. 4 in. wide, if the carpet is 27 in. wide and if $\frac{1}{2}$ ft. must be wasted on each strip to "match the pattern"?

29. Straw matting comes one yard wide. How many yards of plain matting are needed for a room that is 12 ft. wide and 13 ft. long? What will this amount cost at 85¢ per square yard? Make a scale drawing of the room and the strip of matting.

30. What is the value at \$375 per acre of a piece of ground 8 rd. wide and 15 rd. long?

X. TRIANGLES, PARALLELOGRAMS, TRAPEZOIDS

63. Triangles. — The figure at the right is a triangle. It is called **triangle ABC**, or $\triangle ABC$. ($\triangle$ is used to represent the word "triangle.") A triangle has three sides AB, AC, and BC, and three angles $\angle A$, $\angle B$, and $\angle C$. Notice that $\angle A$ is opposite side BC; that $\angle B$ is opposite side AC; and that $\angle C$ is opposite side AB.



EXERCISE 77

1. *a.* Draw a segment AB, 3 inches long.

b. At point A, draw segment AC, making $\angle A$ equal 65°, and side AC, $2\frac{1}{2}$ inches long.

c. Draw BC, and measure it. Print its length on it.

d. Find the sum of the lengths of the sides.

e. Memorize: *The perimeter of a triangle is the sum of the lengths of its sides.*

2. *a.* Draw a segment AB, $3\frac{1}{2}$ inches long. At A, draw AC, making $\angle A$ equal 50°; and at B, draw BC, making $\angle B$ equal 60°.

b. Measure the lengths of sides AC and BC, and find the perimeter of the triangle.

3. What is the perimeter of a triangle if the sides are:

a. 2.5 in., 3.7 in., and 4.2 in. respectively?

b. $4\frac{1}{3}$ *ft.*, $5\frac{1}{4}$ ft., and $6\frac{1}{2}$ ft. respectively?

c. $3\frac{3}{16}$ in., $5\frac{1}{8}$ in., and $4\frac{1}{4}$ in. respectively?

d. 14.3 ft., 10.25 ft., and 12.75 ft. respectively?

e. $5\frac{7}{8}$ in., $7\frac{3}{4}$ in., and $8\frac{1}{2}$ in. respectively?

f. $6\frac{2}{3}$ ft., $7\frac{1}{6}$ ft., and $9\frac{3}{4}$ ft. respectively?

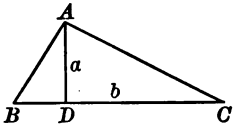
4. *a.* What is the perimeter of a triangle if side AB equals 6 in., BC equals 4 in., and AC equals 5 in.?

b. What per cent of the perimeter is side AC?

c. What per cent of the perimeter is side AB?

d. What per cent of the perimeter is side BC?

5. *a.* Draw triangle ABC, having $BC=4$ inches, $\angle B=70°$, and $AB=3$ inches.



b. Draw AD from A, perpendicular to BC. (See Ex. 6, p. 127.)

c. Along AD, print **altitude**; along BC, print **base.**

d. Memorize: *An altitude of a triangle is a segment drawn from one vertex perpendicular to the opposite side.*

6. *a.* Draw XYZ having $XY=3\frac{1}{2}$ in., $\angle Y=60°$, and $YZ=2\frac{3}{4}$ in.

b. Draw the altitude from Z to XY. Measure it and print its length beside it.

c. Draw the altitude from Y to XZ. Measure it and print its length beside it.

d. Draw the altitude from X to YZ. Measure it and print its length beside it.

e. Do you observe anything about the manner in which the altitudes cross?

7. *a. Draw* any triangle of reasonably large size. *Draw its altitudes.* Observe again the manner in which

the altitudes cross. Are you led to the same conclusion you reached in part *e* of Example 6?

b. After the members of the class have agreed upon a conclusion, write down the conclusion in the following form: *The altitudes of a triangle*, etc.

8. *a*. How many sides of a triangle may be selected as the " base " of the triangle?

b. How many altitudes can be drawn to any one side?

c. How many altitudes does the triangle have all together?

9. *a*. Draw $\triangle MNO$ having $MN=4$ in., $\angle M=55°$, and $\angle N=70°$.

b. Draw the altitude from O to MN, and determine its length.

c. Draw the altitude from N to MO, and determine the length both of it and of MO.

d. Draw the altitude from M to NO, and determine the length both of it and NO. Mark all these lengths along the corresponding segments.

e. Do the altitudes cross as you decided they should in Example 7?

10. *a*. Draw $\triangle ABC$ having $AB=3$ in., $\angle A=110°$, and $AC=2$ in.

b. Draw the altitude from C to AB. It will be necessary to extend AB through point A.

c. Draw the altitude from B to AC.

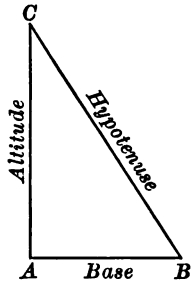
d. Draw the altitude from A to BC.

e. Extend all three altitudes. Do they meet as you *decided they* should, in Example 7?

64. Right triangles.

EXERCISE 78

1. *a.* Draw segment AB 2 inches long. At A, draw segment AC perpendicular to AB, making it 3 inches long. (Remember Ex. 4, p. 126.) Draw segment BC.



b. Along BC, print **hypotenuse**; along AB, **base**; along AC, **altitude.** Below the figure, print **right triangle.**

c. What kind of angles are $\angle B$ and $\angle C$? Measure them, and print inside each the number of degrees there are in it.

d. Complete the following sentences and try to memorize them:

A right triangle is a triangle one of whose angles is a —— *angle.*

The hypotenuse of a right triangle is the side opposite the —— *angle.*

2. *a.* Draw a right triangle whose base is $2\frac{1}{4}$ in., and whose altitude is 3 in.

b. Measure its hypotenuse to sixteenths of an inch. Mark the length on the hypotenuse.

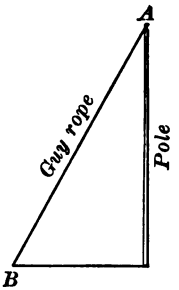
c. What is the perimeter of this triangle?

d. How large is each of the acute angles of this triangle?

3. *a.* Draw right triangle XYZ, having base $XY = 2.75$ in., and altitude $XZ = 3.125$ in.

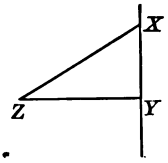
b. Measure its hypotenuse to sixteenths of an inch, *and mark its length* on the hypotenuse.

c. What is the perimeter of this triangle?



4. A pole is 40 feet high. Through its foot, a line 30 feet in length is marked off, at right angles to it.

Make a scale drawing of this figure. Then measure AB, to determine the length of a guy rope running from the top of the pole to point B.



5. In the figure at the left, XZ represents the side view of an awning over a window. XY is 30 in. and YZ is 36 in.

a. Draw triangle XYZ to scale.

b. Measure XZ, and thus determine how many feet of canvas are required for the awning, if 6 inches extra length are purchased for making the awning.

c. What will the canvas cost for 8 such awnings if canvas of the proper width costs \$1.18 per yard of length?

6. A man is building a rectangular gate $4\frac{1}{2}$ ft. high and $3\frac{1}{2}$ ft. wide.

a. Draw to scale the figure representing this gate, letting 1 in. = 1 ft., and then draw one of the diagonals to represent a brace to make the gate keep its shape.

b. How long must this brace be made?

65. Area of a right triangle.

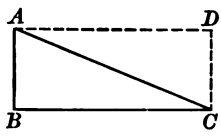
EXERCISE 79

1. *a.* Draw right triangle ABC, having base $BC = 3$ in., and altitude $AB = 2$ in.

b. Draw CD perpendicular to BC at C, making CD 2 in. long. Draw AD.

c. What kind of figure is *ABCD*?

d. How does the amount of surface inside triangle *ABC* compare with the amount inside triangle *ACD*? (Do not merely guess. Make a tracing of triangle *ABC* and compare that with triangle *ACD*.)



e. What is the area of rectangle *ABCD*?

f. What then is the area of triangle *ABC*?

2. What would be the area of the rectangle in Example 1, if the altitude were 5 in. and the base were 4 in.? What then would be the area of triangle *ABC*?

What is the area of the right triangle if:

3. The base is 6 in. and the altitude is 4 in.?

4. The base is 8 ft. and the altitude is 5 ft.?

5. The base is $7\frac{1}{2}$ in. and the altitude is 4 in.?

6. The altitude is $6\frac{2}{3}$ ft. and the base is 9 ft.?

7. The altitude is 5.5 in. and the base is 8 in.?

8. The base is 7.25 ft. and the altitude is 3.5 ft.?

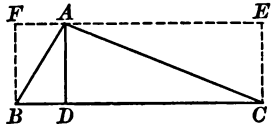
9. The altitude is $15\frac{1}{2}$ ft. and the base is $33\frac{1}{4}$ ft.?

10. The altitude is 24.6 ft. and the base is 65.2 ft.?

66. Area of any triangle.

EXERCISE 80

1. In the figure at the right, *AD* is the **altitude** of triangle *ABC*, and *BC* is the **base** of both triangle *ABC* and rectangle *BCEF*.



a. *What part* of the interior of rectangle *DCEA* is the *interior of triangle ADC*? (Use tracing paper.)

b. What part of rectangle $BDAF$ is triangle BDA?

c. What part then of rectangle $BCEF$ is triangle ABC?

d. Suppose that the altitude AD were 3 in. and the base BC were 8 in. What would be the area of rectangle $BCEF$? What, the area of triangle ABC?

2. If $BC=4$ in. and $AD=2$ in., what is the area of rectangle $BCEF$? Of triangle ABC

3. If $BC=6$ in. and $AD=3$ in., what is the area of rectangle $BCEF$? Of triangle ABC?

4. What is the area of triangle ABC if:

a. Its base is 5 in. and its altitude is 8 in.?

b. Its base is 4 ft. and its altitude is 7 ft.?

c. Its altitude is 3.5 yd. and its base is 6 yd.?

The foregoing examples give the

Rule. — The area of a triangle is one half the product of the base and the altitude of the triangle.

The formula for the area of a triangle is

$$A=\tfrac{1}{2}\,ab$$

where A represents the number of square units of surface, a the number of linear units in the altitude, and b the number of the *same kind* of linear units in the base.

Example. — What is the area of a triangle whose base is 8 ft. 6 in. and whose altitude is $1\frac{1}{2}$ yd.?

Solution. — 1. The formula is $A=\frac{1}{2}\,ab$.

2. In this example, $a=1\frac{1}{2}$ yd. $=4.5$ ft.
$b=8$ ft. 6 in. $=8.5$ ft.

3. Therefore $A=\frac{1}{2}\times 4.5\times 8.5$

4. or $A=19.125$ sq. ft.

$$\begin{array}{r} 4.5 \\ 8.5 \\ \hline 22.5 \\ 360 \\ \hline 2\overline{)38.25} \\ 19.125 \end{array}$$

EXERCISE 81

Find the area of a triangle, when:

1. $a = 7$ in., and $b = 13.5$ in.

2. $b = 6.5$ ft., and $a = 2$ ft. 3 in.

3. $a = 18$ rd., and $b = 6\frac{2}{3}$ rd.

4. $a = 11$ yd. 1 ft., and $b = 6$ yd. 2 ft.

5. $b = 24.8$ ft., and $a = 15$ ft. 6 in.

6. *a.* Draw a triangle ABC having $AB = 4\frac{1}{2}$ in., $\angle B = 50°$, and $\angle A = 65°$.

b. Draw the altitude to side AB, and determine its length.

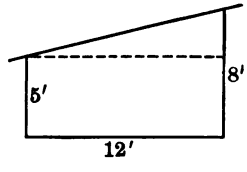
c. What is the area of this triangle?

7. *a.* Draw a triangle XZY having $XY = 3.75$ in., $\angle X = 70°$, and $XZ = 2.5$ in.

b. Draw the altitude from Z to XY, and determine its length. Then find the area of the triangle.

c. Now draw the altitude from Y to XZ, and determine its length. Again find the area of the triangle. While you will not obtain exactly the same result, you should obtain two results for the area that are approximately equal.

8. *a.* At the right is the end view of a chicken house. The dotted line shows how this surface can be divided into two figures whose areas can be computed. Using the dimensions given in the figure, find the area of the end of the building.



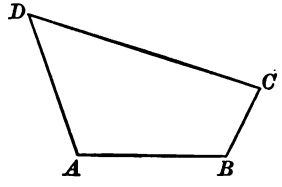
b. Suppose that the house is 16 ft. long. Find the *area of the back* wall and also of the front wall. (It

would be well to make a scale drawing of each of these walls.)

c. What is the cost of painting the four walls at 27¢ per sq. yd.? (While the front will consist of windows, it costs as much to paint the division strips of a window as it does to paint a surface as large as that covered by the window.)

NOTE. — If any member of the class, or if the class together, can actually measure a building of this shape, it may be more interesting to use the measurements so obtained.

9. *a.* Draw a segment AB 3 in. long. At A, draw AD, making $\angle A = 105°$, and AD $2\frac{1}{2}$ in. long. At B, draw BC, making $\angle B = 100°$, and BC 3.5 in. long. Then draw CD.



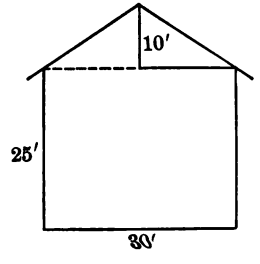
b. Now draw BD and determine its length.

c. Draw the altitude of triangle ABD from A. Measure it. Find the area of triangle ABD.

d. Draw the altitude of triangle BCD from C. Measure it. Find the area of triangle BCD.

e. What then is the area of $ABCD$?

10. *a.* Find the area of one end wall of the barn pictured at the right.



b. If the barn is 60 ft. long, find the total area of the two end walls and the two side walls.

c. Find the cost of painting the walls at 28¢ per square yard.

NOTE. — *See the* Note to Ex. 8, above.

Supplementary Topics

(Compasses are needed for some of this work.)

67. Triangles in general.

EXERCISE 82

1. *a.* Draw any triangle ABC of reasonably large size.

b. Measure its sides to sixteenths of an inch.

c. Is $AB+BC$ more or less than AC?

d. Is $AB+AC$ more or less than BC?

e. Is $BC+AC$ more or less than AB?

f. Below, write: *The sum of two sides of a* —— *is* —— *than the third side.*

2. *a.* Draw a large triangle XYZ. Measure each of its angles. Find the sum of these measures.

b. Draw a second triangle, and repeat part *a.*

c. What seems to be true?

d. Below, write: *The sum of the angles of a* —— *is* —— *degrees.*

3. If one angle of a triangle measures 90°, and a second angle measures 40°, what is the measure of the third angle?

4. Tell the size of the third angle of a triangle if:

a. One angle = 45° and a second = 65°;

b. One angle = 53° and a second = 49°;

c. One angle = 28° and a second = 67°.

5. If one angle of a triangle measures 90°, how great is the sum of the other two angles?

6. If one angle of a triangle is 90°, what kind of angles *must each of the* others be?

7. If $\angle A = 90°$, and $\angle B = 30°$:

a. How large is $\angle C$?

b. What per cent of $\angle A$ is $\angle C$?

c. What per cent of $\angle B$ is $\angle C$?

d. What per cent of $\angle A$ is $\angle B$?

8. *a.* If $\angle A = 40°$, and $\angle B = 60°$, what does $\angle C$ equal?

b. What per cent of $\angle B$ is $\angle A$? is $\angle C$?

c. What per cent of $\angle C$ is $\angle B$? is $\angle A$?

d. What per cent of $\angle A$ is $\angle B$? is $\angle C$?

9. If one angle of a triangle equals 80°, and the other two are equal to each other, how large is each of them?

10. If the three angles of a triangle are all equal, how large is each of them?

68. Constructing triangles with compasses and rules. If the meaning of *circle*, *radius*, and *arc* as given in section 73, p. 176, have not been learned by the class, these should be studied now.

EXERCISE 83

1. Constructing a triangle with sides 2.5 in., 2 in., and $1\frac{3}{4}$ in. in length.

a. Draw segment AB 2.5 in. long.

b. With radius of 2 in., place the point of the compass on A, and draw an arc above segment AB.

c. With radius of $1\frac{3}{4}$ in., place the point of the compasses on B, and draw a second arc, cutting the first arc at point C.

d. Draw AC and BC. Then triangle ABC has the required sides.

2. Construct triangle XYZ having $XY=3$ in., $XZ=1.5$ in., and $YZ=2.5$ in.

3. *a.* Construct triangle RST having $RS=4$ in., $ST=3.25$ in., and $RT=2.75$ in.

b. Draw the altitude from vertex T, and determine its length.

c. Measure angles R and S.

4. *a.* Construct very carefully triangle ABC having $AB=4$ in., $BC=3$ in., and $AC=2.5$ in.

b. Then construct a second triangle having the same sides.

c. Now make a tracing of the first triangle, and compare that with the second triangle. What do you discover?

d. Also compare the tracing of your triangle with the triangles made by one or more of the other pupils of your class. What seems to be true about all of the triangles?

5. Try to construct a triangle whose sides measure 4 in., 2 in., and 1.5 in. respectively.

6. Try to construct a triangle whose sides measure 4.5 in., 2.75 in., and 1.75 in. respectively.

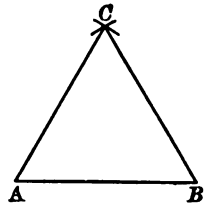
69. Isosceles and equilateral triangles.

EXERCISE 84

1. *a.* Construct triangle ABC having $AB=2$ in., $AC=2.75$ in., and $BC=2.75$ in.

b. On AB, print **base**; on AC and BC, print **side**.

c. Below, print: **Isosceles triangle.** *An isosceles triangle has two equal sides.*



2. *a.* Draw an isosceles triangle XYZ having the base $XZ = 3$ in., and the equal sides XY and ZY each $= 2\frac{1}{4}$ in.

b. Measure and compare the angles X and Z.

3. *a.* Draw the isosceles triangle HKL with base $HK = 2.5$ in., and equal sides HL and KL each $= 3$ in.

b. Compare the angles H and K. What seems to be true?

c. Below, write: *If a triangle is isosceles, the angles opposite the equal sides are* ——.

4. *a.* Do you see any isosceles triangles in your classroom?

b. Can you see any isosceles triangles from the windows of your classroom? If you can find any in pictures, measure the angles opposite the equal sides.

5. *a.* Draw segment RS, 2.5 in. long.

b. At its center T, draw segment TU perpendicular to RS, making TU any length.

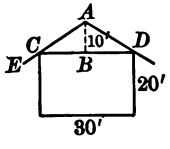
c. Then draw RU and SU.

d. What kind of triangle is $\triangle RSU$? Test its sides.

e. Along TU, print **altitude**. Along RS, print **base**.

6. The end of a gable roof has the form of an isosceles triangle.

a. Make a scale drawing of the end of the barn pictured at the right.



b. Measure the angles ACD and ADC.

c. How long would the rafter AC be?

NOTE. — See the Note, Ex. 8, p. 159.

7. *a.* Construct the triangle whose three sides each are *2.5 in. long.*

b. Below, print: **Equilateral triangle.** *An equilateral triangle has three equal sides.*

c. Measure its three angles. What seems to be true?

8. *a.* Construct the equilateral triangle whose sides measure 3 inches.

b. Measure and compare its angles. What seems to be true?

c. Below, write: *The angles of an equilateral —— are all ——.*

9. How large must each angle of every equilateral triangle be?

10. *a.* Draw the equilateral triangle whose sides each measure 2.875 in.

b. What is its perimeter?

c. Draw and find the length of each of its altitudes.

d. What seems to be true?

70. Parallelograms.

The figure at the right is a **parallelogram.**

EXERCISE 85

1. Compare the lengths of sides AB and CD; also of sides AD and BC.

2. What kind of lines are AB and CD? AD and BC?

3. *a.* Compare $\angle A$ and $\angle C$ by means of tracing paper; also compare $\angle B$ and $\angle D$.

b. Measure $\angle A$ and $\angle B$ with your protractor. Find $\angle A + \angle B$.

4. *a.* Draw segment AB 4 in. long.

b. *At A, draw* AD making $\angle A = 70°$, and $AD = 2$ in.

c. At B, draw BC, making $\angle ABC = 110°$, and $BC = 2$ in.

d. Draw CD.

e. What kind of figure have you drawn? (If your figure is not well drawn, draw it again.)

f. Complete the following sentences, supplying the missing words:

A parallelogram has (how many) *sides. Its opposite sides are* ——; *they are also* —— *in length. The opposite angles are* ——. *The sum of the two angles at the ends of one side is* —— *degrees.*

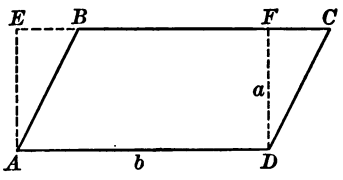
5. *a.* Draw parallelogram $ABCD$ having $\angle A = 60°$, $AD = 3\frac{1}{2}$ in., and $AB = 2$ in.

b. From D, draw DE perpendicular to AB. (See Ex. 6, p. 127.) Beside it, print **altitude**; along AB, print **base.**

c. Measure DE.

d. From C, draw CF perpendicular to AB extended (made longer). Measure it, also. How does it compare with DE?

6. *a.* Draw parallelogram $ABCD$, having base $AD = 4$ in., $\angle A = 60°$, and $AB = 2$ in.



b. From D, draw DF perpendicular to BC.

c. From A, draw AE perpendicular to BC, extended.

d. Make a tracing of $\triangle ABE$. Compare it with $\triangle DCF$. How does the amount of surface inside $\triangle ABE$ compare with the amount inside $\triangle DCF$?

e. How does the amount of surface inside rectangle $ADFE$ compare with the amount inside parallelogram $ABCD$?

f. Measure DF and AE. Find the area of $ADFE$.

g. What then is the area of parallelogram $ABCD$?

7. If in Example 6, AD had been 6 in., and DF 2 in., what would have been the area of rectangle $ADFE$? Of the parallelogram?

Give the area of the parallelogram if:

8. Base $AD = 12$ in., and altitude $DF = 4$ in.

9. Base $AD = 16$ yd., and altitude $DF = 3$ yd.

10. Base $AD = 25$ ft., and altitude $DF = 7$ ft.

11. If the altitude $= 8$ ft., and the base $= 12\frac{1}{2}$ ft.

12. If the altitude $= 9$ in., and the base $= 16\frac{2}{3}$ in.

The previous examples give the

Rule. — The area of a parallelogram equals the product of the base and the altitude of the parallelogram.

The formula for the area of a parallelogram is

$$A = ab$$

where A = the number of units of surface, a = the number of linear units in the altitude, and b = the number of the same linear units in the base.

Example. — What is the area of a parallelogram whose base is 11 ft. 4 in. and whose altitude is $1\frac{1}{6}$ yd.?

Solution. — 1. The formula for the area of a parallelogram is $A = ab$.

2. In this example, $a = 11$ ft. 4 in. $= 11\frac{1}{3}$ ft.
and $b = 1\frac{1}{6}$ yd. $= 3\frac{1}{2}$ ft.

3. Then $A = 11\frac{1}{3} \times 3\frac{1}{2}$

4. $= \frac{34}{3} \times \frac{7}{2}$

5. $= \frac{119}{3} = 39\frac{2}{3}$ sq. ft.

EXERCISE 86

Find the area of the parallelogram whose:

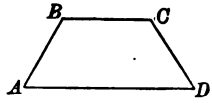
1. Base = 42 in. and altitude = 13 in.
2. Base = 16 rd. and altitude = $5\frac{1}{2}$ rd.
3. Base = 33 ft. 6 in. and altitude = 15 ft.
4. Altitude = $2\frac{1}{4}$ in. and base = $8\frac{1}{2}$ in.
5. Altitude = 11.25 in. and base = 18.6 in.

Find A when:

6. $a = 24.8$ rd.; $b = 16.5$ rd.
7. $b = 3.25$ ft.; $a = 12$ yd.
8. $a = 8\frac{1}{3}$ yd.; $b = 1.5$ yd.
9. $a = 6$ ft. 4 in.; $b = 11$ ft. 3 in.
10. $b = 5$ yd. 2 ft.; $a = 6$ yd. 1 ft.
11. $a = 16$ ft. 8 in.; $b = 24$ ft. 6 in.

71. Trapezoids.

The figure at the right is a **trapezoid.**



EXERCISE 87

1. *a.* What kind of lines are AD and BC?

b. Are AB and CD the same sort of lines?

c. Measure $\angle A$ and $\angle B$, and find $\angle A + \angle B$.

2. *a.* Draw segment AD 4 inches long.

b. Draw AB, making $\angle A = 60°$, and $AB = 2$ inches.

c. Draw BC, making $\angle ABC = 120°$, and $BC = 1\frac{1}{2}$ inches.

d. Draw DC. What kind of figure have you drawn?

(If it is not well drawn, draw it again.)

e. Measure $\angle D$ and $\angle C$, and find $\angle D + \angle C$.

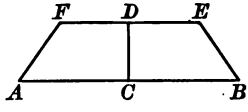
f. Along AD, print **lower base**; along BC, **upper base**; along AB, **side**; along CD, **side.**

g. From B, draw BE perpendicular to AD. Along it, print **altitude.**

h. Complete the following sentences:

A trapezoid has —— sides. One pair of opposite sides are ——; the other pair are not. One of the parallel sides is called the —— —— base; the other is called the —— ——. A line drawn from a point of one base perpendicular to the other —— is called the ——.

3. *a.* Draw segment $AB = 3\frac{1}{2}$ in. At its mid-point C, draw CD perpendicular to AB, making CD $1\frac{1}{2}$ in. long. At D, draw EF perpendicular to CD, making FD and DE each 1 in. long. Draw AF and EB.



b. Compare the lengths of AF and EB.

c. Compare the angles A and B; also compare the angles E and F.

d. Below, print or write: $ABEF$ is an **isosceles trapezoid.**

e. Complete the following sentences:

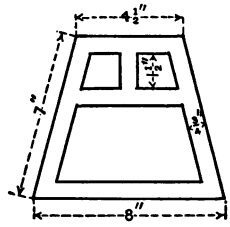
Its two sides that are not parallel are ——. The angles at the ends of the lower —— are ——; also the angles at the ends of the —— base are ——.

4. *a.* Draw an isosceles trapezoid whose lower base is 2.75 in.; whose altitude is 1.875 in.; and whose upper base is 2.25 in.

b. Draw the two diagonals of this trapezoid. Measure *each of them*, and print its length upon it. What do you *discover?*

5. Can you find any trapezoids in your classroom? Can you see any from the windows of your classroom? What kind of trapezoids seem most common?

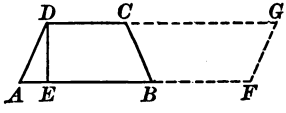
6. At the right is a sketch of one side of a hall lantern. Draw this figure half size.



72. Area of a trapezoid.

EXERCISE 88

1. *a.* Draw trapezoid $ABCD$, having $AB = 2\frac{1}{2}$ in.; altitude $DE = 2$ in.; and $CD = 1\frac{1}{2}$ in.



b. Extend AB to F, making $BF = CD$.

c. Extend DC to G, making $CG = AB$.

d. Draw FG. What kind of figure is $AFGD$?

e. By means of tracing paper, compare trapezoid $ABCD$ with the figure $BFGC$. What do you discover?

f. What is the area of $AFGD$?

g. What then is the area of $ABCD$?

2. In the preceding example, suppose that $AB = 5$ in., $CD = 3$ in., and $DE = 2$ in.

a. What is the area of the parallelogram?

b. Of the trapezoid?

3. Give the area of the parallelogram and of the trapezoid if $AB = 3$ in., $CD = 2$ in., and $DE = 4$ in.

4. Give the area of the trapezoid alone if $AB = 4\frac{1}{2}$ in., $CD = 2\frac{1}{2}$ in., and $DE = 3$ in.

5. Give the area of the trapezoid if the lower base = 6 in., the upper base = 3 in., and the altitude = 3 in.

The foregoing examples suggest the

Rule. — The area of a trapezoid equals one half the product obtained by multiplying the sum of the bases by the altitude.

The formula for the area of a trapezoid is

$$A = \tfrac{1}{2}\, a(b+c)$$

where A = the number of units of surface, a = the number of linear units in the altitude, b = the number of the *same* linear units in the lower base, and c = the number of the *same* linear units in the upper base.

The **parentheses**, ( ), mean that the numbers b and c must be added before multiplying by a.

Example. — Find the area of the trapezoid whose altitude is 4 ft. 3 in., whose lower base is 1 yd., and whose upper base is 1 ft. 6 in.

Solution. — 1. The formula for the area of a trapezoid is $A = \tfrac{1}{2}\, a(b+c)$.

2. In this example, $a = 4$ ft. 3 in. $= 4\tfrac{1}{4}$ ft.
$b = 1$ yd. $= 3$ ft.
$c = 1$ ft. 6 in. $= 1\tfrac{1}{2}$ ft.

3. Therefore $A = \tfrac{1}{2} \times 4\tfrac{1}{4} \times (1\tfrac{1}{2} + 3)$

4. $= \tfrac{1}{2} \times \tfrac{17}{4} \times \tfrac{9}{2}$

5. $= \tfrac{153}{16} = 9\tfrac{9}{16}$ sq. ft.

EXERCISE 89

Find the area of the trapezoid when:

1. The altitude is 5 in., the lower base is 11 in., and the upper base is 9 in.

2. The lower base is 16 ft., the altitude is 8 ft., and the *upper base is 4 ft.*

3. The upper base is 12 yd., the lower base is 5 yd., and the altitude is 4 yd.

4. $a = 5$ ft., $b = 6\frac{1}{3}$ ft., and $c = 5\frac{2}{3}$ ft.

5. $b = 11\frac{1}{2}$ ft., $c = 8\frac{1}{2}$ ft., and $a = 6$ ft.

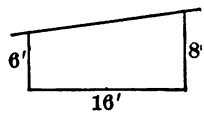
6. $c = 3\frac{1}{4}$ in., $b = 7\frac{1}{2}$ in., and $a = 4$ in.

7. $a = 2.3$ in., $c = 7.5$ in., and $b = 9.4$ in.

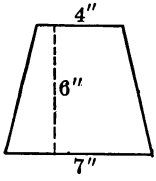
8. $b = 13.4$ in., $c = 6.8$ in., and $a = 5.3$ in.

9. $c = 4$ ft. 3 in., $a = 3$ ft., and $b = 6$ ft. 6 in.

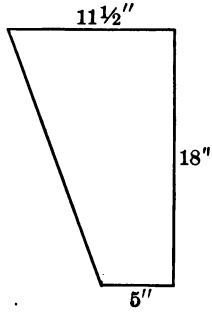
10. $a = 5$ ft. 6 in., $c = 8$ ft. 4 in., and $b = 11$ ft. 8 in.



11. At the right is pictured the end of a shed. Find the area of the end of the building.

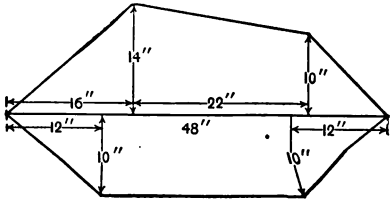


12. At the left is the plan for one of six faces of a lamp shade. Find the area of the six faces.



13. At the right is a scale drawing of one piece of plate glass used in a winter top for a certain automobile. How many square inches are there in the piece?

14. Find the area of the figure represented by the scale drawing below.



EXERCISE 90

Review of Rectangles

1. Draw a rectangle whose base is $3\frac{1}{2}$ in. and altitude $2\frac{1}{2}$ in.

2. Draw the diagonals of this rectangle, and measure them.

3. What do you know about the diagonals of a rectangle?

4. Draw a square whose sides measure 3.75 in.

5. Draw its diagonals.

6. What do you know about the diagonals of a square?

7. Draw to scale a rectangle 75 rd. wide and 120 rd. long. Measure one of its diagonals, and determine its real length.

8. What is the rule for finding the area of a rectangle? What is the formula?

9. What is the area of the rectangle whose base is 15.5 ft. and altitude 6.2 ft.?

10. What is the area of the square whose side is 6.4 in.?

11. What is the cost at 25¢ per square foot of a cement floor 15 ft. by 18.5 ft.?

12. How many acres are there in a field that is 30 rd. long and 25 rd. wide?

13. What is the cost of plastering a ceiling which is 13 *ft. 6 in. long and* 9 ft. 6 in. wide at 40¢ per square yard?

EXERCISE 91

Review of Triangles

1. Draw a triangle. Mark its base and the altitude to that base.

2. What is the perimeter of the triangle whose sides are $4\frac{1}{2}$ in., $3\frac{1}{2}$ in., and 3 in. respectively?

3. What is the area of a triangle whose base is 5 ft. and altitude is 6 ft.?

4. Draw the triangle ABC, when $AB = 2\frac{1}{2}$ in., $\angle A = 60°$, and $AC = 3$ in. Measure BC.

5. What is the perimeter of a triangle?

6. What is the rule for finding the area of a triangle?

7. What is the formula for the area of a triangle?

8. What is the area of a triangle when $a = 6.5$ in. and $b = 11.5$ in.?

9. *a.* Draw $\triangle ABC$, having $AB = 3\frac{1}{2}$ in., $\angle A = 70°$, and $AC = 2\frac{1}{2}$ in.

b. Measure BC and find the perimeter.

c. Draw the altitude to side AB. Measure it.

d. Find the area of the triangle.

10. Draw the right triangle whose base is 4 in. and altitude is $2\frac{1}{2}$ in. Find the length of its hypotenuse.

EXERCISE 92

Review of Supplementary Work on Triangles

1. With compasses, construct the triangle whose sides are 2 in., $2\frac{1}{4}$ in., and 3 in. What is its perimeter?

2. Construct the equilateral triangle whose sides are *2.75 in. long.* What is its perimeter?

3. Construct the isosceles triangle whose equal sides are each 2 in. long and whose other side is $1\frac{1}{2}$ in. long. What is its perimeter?

4. If one angle of a triangle contains 35°, a second contains 65°, how many degrees does the third one contain?

5. Draw to scale the triangle whose sides measure 25 ft., 30 ft., and 40 ft.

EXERCISE 93

Review of Parallelograms

1. Draw a parallelogram. Mark its base and altitude.

2. What do you know about the sides of a parallelogram?

3. What do you know about the angles of a parallelogram?

4. Draw parallelogram $ABCD$, having $AB=3$ in., $\angle B=50°$, $BC=2$ in., $\angle C=130°$.

5. What is the area of the parallelogram whose base is 10 ft. and altitude is 7.5 ft.?

6. What is the rule for finding the area of a parallelogram?

7. What is the formula for the area of a parallelogram?

8. When $a=11$ ft. and $b=15$ ft., what is the area of the parallelogram?

9. Obtain the area of the parallelogram whose base is 6 ft. 3 in. and altitude 2 ft. 8 in.

10. Find the area of the parallelogram when $a=4.3$ ft. and $b=21.4$ ft.

EXERCISE 94

Review of Trapezoids

1. Draw a trapezoid. Mark its lower base; its upper base; its sides. Draw its altitude.

2. What is an isosceles trapezoid?

3. *a.* Draw the isosceles trapezoid whose lower base is 4 in., whose altitude is 2 in., and whose upper base is 3 in.

b. What is the area of this trapezoid?

4. What is the rule for finding the area of any trapezoid?

5. What is the area of the trapezoid whose lower base is 11.5 in., upper base 7.8 in., and altitude 6.3 in.?

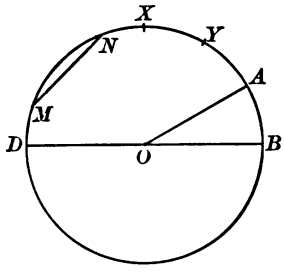
6. What is the formula for the area of a trapezoid?

7. Find the area of the trapezoid in which $a = 5.5$ in., $b = 12.25$ in., and $c = 9.75$ in.

8. Draw a trapezoid whose lower base AB is 3.75 in., whose $\angle A$ is 75°, whose $\angle B$ is 85°, and whose altitude drawn to side AB is 2 in.

XI. CIRCLES

73. The curved line at the left is a **circle**. Point O is its **center**; OA is a **radius** of the circle (plural, **radii**); BD is a **diameter**; the part XY is an **arc** of the circle; segment MN is a **chord**.



A circle is drawn by means of compasses.

NOTE. — In older texts, the curved line was called the *circumference* of the circle, and the portion of the plane inside it was called the *circle*. Best usage favors the meanings given in this text. These meanings are as easily learned as are less correct ones. For *circumference*, see § 74.

EXERCISE 95

1. *a.* A radius is what part of a diameter?

b. How many times a radius is a diameter?

2. *a.* Draw a circle with radius 1 inch. Inside it, draw three radii.

b. Draw two diameters.

c. How many radii can be drawn? How many diameters?

3. *a.* From a center O, draw a circle with radius $1\frac{1}{4}$ in.

b. From the same center, draw a circle with radius $1\frac{1}{2}$ in.; one with radius $1\frac{3}{4}$ in.

c. Below, print: **Concentric circles.** This means, circles having the same center.

4. Draw a circle of radius $1\frac{1}{2}$ inches. On it, mark a point by the letter A. Through A, draw a chord of the circle; then draw a second chord. Then draw the longest chord. What other name does this longest chord have?

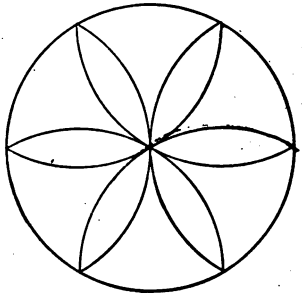
5. Draw a circle of radius $1\frac{1}{2}$ inches. Draw any diameter of it. With tracing paper, copy one of the two parts into which the circle is divided. Compare this part with the other part of the circle. What do you discover?

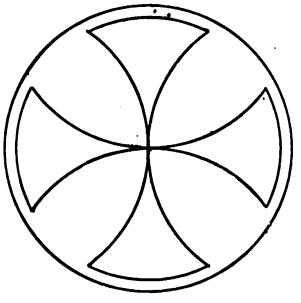
Below, write: *A diameter of a circle divides the circle into two —— parts.* Each of these parts is called a **semicircle.**

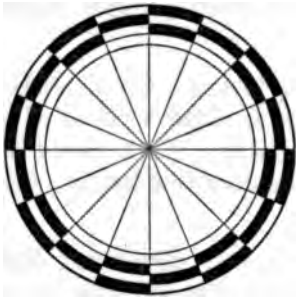
6. *a.* Draw a circle of radius $1\frac{1}{2}$ inches. Through its center O, draw a diameter AB; also draw a diameter CD which is perpendicular to AB.

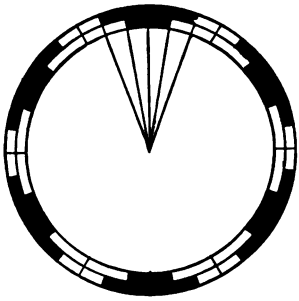
b. These two diameters divide the circle into four parts. Compare these four arcs by means of tracing paper.

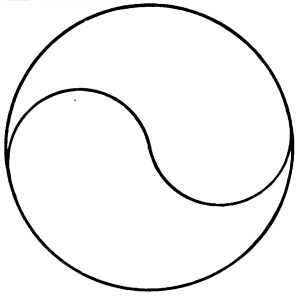
7. Try to copy some of the following circle designs. You will get better results if you make your figures large.





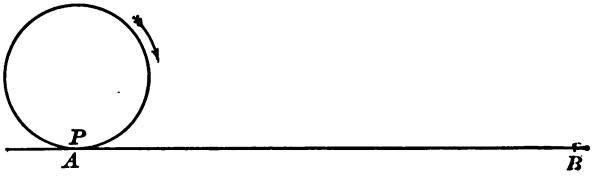






74. The length of a circle can be determined approximately as follows:

On the circle mark a point P; place this point over a point A of a straight line. Roll the circle, without allowing it to slip, along the straight line until the point P again falls on the line. Mark this last point B. Measure the segment AB. Its length is approximately equal to that of the circle.



Circumference of a circle means the same as length of the circle.

EXERCISE 96

1. The pupils of one class were directed to measure the length and the diameter of a circular object at home; they were also told to divide the length of the circle by that of its diameter. The following measurements were accepted in class as being sufficiently accurate:

Object	Length of Diameter	Length of Circle	Circle Divided by Diameter
Lamp base	$5\frac{1}{2}$ in.	$17\frac{1}{4}$ in.	?
Jardinière	$8\frac{1}{16}$ in.	$25\frac{7}{16}$ in.	?
Plate	$9\frac{7}{16}$ in.	$29\frac{3}{8}$ in.	?
Pail	$6\frac{1}{16}$ in.	$19\frac{9}{16}$ in.	?

a. Find the quotients that should appear in the fourth column, carrying out the division to two decimal places.

b. Find the average of these quotients.

In senior high school geometry, *it is proved that the quotient obtained by dividing the length of a circle by the length of its diameter is the same for all circles.* Since the measurements given above were only approximately correct, the quotients obtained are only approximately equal. However, the average of these quotients is the correct result to two decimal places, for it is proved in senior high school geometry that

the length of a circle divided by the length of its diameter $= 3.14159^{+}$.

For ordinary computation, the value 3.1416 is used, so that

the length of a circle $= 3.1416 \times$ *the length of its diameter.*

The number 3.14159+ is represented in mathematics by the Greek letter π (read pī), corresponding to our letter *p*.

If C = the length of the circle, D = the length of its diameter, the formula for the length of the circle is

$$C = \pi D, \text{ or } C = 2\pi R$$

since $D = 2 \times R$.

NOTE. — Another value of the number π that is often used is $3\frac{1}{7}$, so that $C = 3\frac{1}{7} \times D$ or $C = 2 \times 3\frac{1}{7} \times R$.

Example. — Find the length of the circle whose radius is 11.5 in.

Solution I. — 1. Use the formula $C = 2\pi R$, letting $\pi = 3.1416$.

2. Then $C = 2 \times 3.1416 \times 11.5$
3. $= 6.2832 \times 11.5$
4. $= 72.2568$
5. $= 72.2+$

```
6.2832
  11.5
314160
62832
62832
72.25680
```

NOTE. — Since the radius is given correct to only one decimal place, the length is written correct to only one decimal place.

Solution II. — 1. Use the formula $C = 2\pi R$, letting $\pi = 3\frac{1}{7}$.

2. Then $C = 2 \times 3\frac{1}{7} \times 11.5$
3. $= 2 \times \frac{22}{7} \times \frac{23}{2}$
4. $= \frac{506}{7} = 72.285$
5. Again $C = 72.2+$

EXERCISE 97

1. *a.* Measure the length and the diameter of the circle bounding some circular object, — as large an object as convenient.

b. Divide the length of the circle by the length of its diameter, carrying the result out to two decimal places.

c. Bring the result to class to-morrow, at which time the average of the reasonably accurate results can be found.

2. Find the decimal value of $3\frac{1}{7}$ correct to four decimal places, and compare the result with 3.1416.

Find the lengths or circumferences of the circles whose diameters are given below, using 3.1416 *in the even* examples and $3\frac{1}{7}$ *in the odd* examples.

3. 6 in. 5. 15 ft. 7. $2\frac{1}{3}$ yd. 9. $16\frac{1}{2}$ ft. 11. $25\frac{1}{2}$ in.

4. 10 in. 6. $8\frac{1}{2}$ in. 8. 5.25 ft. 10. 3.75 ft. 12. $33\frac{1}{3}$ in.

Find the circumference when the radius is:

13. 7.5 in. 15. 2.6 yd. 17. 13.16 ft. 19. 18.375 in.

14. 11.28 ft. 16. 11.8 in. 18. 4.625 ft. 20. 15.83 in.

21. What is the circumference of a 33-inch auto tire? Of a 26-in. bicycle tire?

22. How many times must an auto wheel with a 30-inch tire revolve in going one mile?

23. A lamp shade frame is 16 inches in diameter on the lower edge and 9 inches in diameter on the upper edge. How long must a piece of silk be in order to reach around the lower edge of the frame?

24. The diameter of the earth is approximately 7912 miles. What is the approximate length of the equator?

25. A man wants to cover a "round" furnace pipe with some asbestos paper. The pipe is 12 inches in diameter. How long must the paper be to go around the pipe once and lap over 1 inch?

26. A boy has a circular hover for his small chickens. The diameter is 21.5 inches. He wants enough flannelet

to make a curtain to hang from the base of the hood. How long must the cloth be to reach around the hood?

27. Find the diameter of the circle whose circumference is:

a. 28.2744; *b.* 20.4204; *c.* 47.124. (Use $\pi = 3.1416$.)

28. Find the radius of the circle whose circumference is:

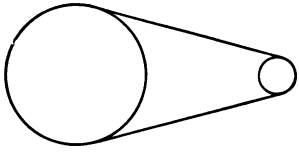
a. 26 in.; *b.* 375 ft.; *c.* 100 in. (Use $\pi = 3.1416$.)

29. *a.* What must be the diameter of a "quarter mile" circular running track, at its inside edge? (Use $\pi = 3\frac{1}{7}$.)

b. If the track is made 25 feet wide, what is the distance around it at its outside edge? How many more feet is this than the distance around the inside edge?

c. What is the distance around the track along the center of the track?

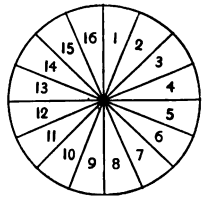
30. The diameter of a large pulley wheel on a gas engine is 18 inches; the diameter of the wheel on a vacuum cleaner run by the engine is 5 inches.



a. What is the circumference of the engine wheel?

b. What is the circumference of the smaller wheel?

c. If the belt does not slip, how many times does the small wheel revolve while the large wheel is revolving once. (Divide the greater circumference by the smaller.)



75. Rule for the area of a circle. By drawing radii, the interior of a circle can be divided into sixteen (or more) equal parts. Each part is approximately triangular in shape, — the base being a small arc of the circle, and the altitude a radius of the circle.

The area of each part, then, is approximately one half the radius multiplied by $\frac{1}{16}$ of the circumference. When the areas of all the parts are added, thus obtaining the area of the circle, then

Rule. — The area of a circle equals one half the radius multiplied by the circumference of the circle.

EXERCISE 98

Using $3\frac{1}{7}$ for π in the odd examples, and 3.1416 in the even examples, find the area of the circle whose:

1. Diameter is 16 in.
2. Diameter is 20 in.
3. Diameter is 3.5 in.
4. Radius is 12 in.
5. Radius is 25 in.
6. Radius is 6.2 in.

7. The radius of one circle is 5 ft.; that of a second circle is 10 ft.

a. Compare their areas.

b. Compare their circumferences.

8. The side of a square is 8 ft. The diameter of a circle is 8 ft. Which has the greater area and how much?

9. The diameter of a Ford piston is $3\frac{3}{4}$ inches. What is the area of the surface of the piston?

10. The radius of the piston of a steam engine is $7\frac{1}{2}$ inches.

a. Find the area of the surface of the piston.

b. What is the total pressure on the piston if the steam pressure is 110 lb. per square inch?

11. How many square miles of territory are included in a circle one mile in radius, whose center is at the " center " of a town?

12. Over how many square yards of ground can a horse graze if it is tied to a stake by a rope 25 ft. long?

13. Find the area of the ring lying between two circles drawn from the same center, whose radii are 8 inches and 10 inches respectively.

14. *a.* A circular flower bed is 11 ft. in diameter. How many square feet of surface lie inside it?

b. If the bed is to be planted to tulips, and each tulip is allowed 36 square inches of surface, how many tulips will be needed to plant the bed, — approximately?

15. What is the cost of coating with bronze a circular ceiling plate of an electric light fixture, 16 inches in diameter, if the cost is $\frac{3}{4}¢$ per square inch?

16. If the circumference of a circle is 88 inches, what is the area? (Use $\pi = 3\frac{1}{7}$.)

17. A man planned a circular flower bed to be filled with geraniums. He wants the diameter to be 10 feet, and, to estimate the number of plants required, allows each plant 1.5 square feet. What is the approximate number of plants needed? (Use $\pi = 3\frac{1}{7}$.)

18. A man planned to make some circular rings to be used on stakes to support plants in a flower garden. Besides the wire actually used in the circular ring, about five inches more is required for each ring to attach it to the stake. How much wire must he get for 50 rings 15 inches in diameter, 100 rings 18 inches in diameter, and 75 rings 21 inches in diameter? ($\pi = 3\frac{1}{7}$.)

19. What is the cost of grinding the edge of a circular glass 9 inches in diameter, at 3¢ per inch?

20. *In the city of Indianapolis is a monument erected on a circular plot of ground, whose diameter is 342 feet.*

Around this plot of ground is a sidewalk 25 feet wide; outside the sidewalk is a street 45 feet wide.

Find:

a. The length of the inside edge of the sidewalk.

b. The area of the sidewalk.

c. The area of the street.

d. The cost of resurfacing the street at \$3.25 per square yard.

e. The present cost of a new sidewalk at 21¢ per square foot.

76. Squaring a number is necessary in the next formula to be studied. When a number is multiplied by itself, the square of the number is obtained.

Thus, 3×3 or 9 is the square of 3. It is indicated thus: 3^2. The small number 2, at the right and a little above the number 3, is called an **exponent**. 3 is called the **base**. The exponent 2 means that two threes are to be multiplied together.

Similarly 5^2 means 5×5 or 25; 7^2 means 7×7 or 49.

EXERCISE 99

Find the numerical value of:

1. 4^2	4. 10^2	7. 14^2	10. 6.5^2	13. 2.25^2
2. 6^2	5. 12^2	8. 15^2	11. 3.2^2	14. 4.07^2
3. 8^2	6. 11^2	9. 28^2	12. 7.3^2	15. 5.43^2

77. Formula for the area of a circle.

The area of a circle $= \frac{1}{2}$ radius $\times$ circumference.

If R *represents* the radius, then the circumference is $2\pi R$ *(p. 180).*

Therefore the area $= \frac{1}{2} R \times 2\pi R$

$= \pi R \times R = \pi R^2$.

If A represents the area of the circle, then the formula is

$$A = \pi R^2$$

This formula can be expressed by the

Rule. — The area of a circle is π times the square of the radius.

Example. — Find the area of the circle whose diameter is 26 inches.

Solution. — 1. The formula is $A = \pi R^2$.

2. Since the diameter is 26, then the radius is 13.

3. Therefore $A = \pi \times 13^2$

$= 3.1416 \times 169$

$= 530.9+$ sq. in.

13	3.1416
13	169
39	282744
13	188496
169	31416
	530.9304

EXERCISE 100

Find the area of the circle when:

1. The radius is 8 in.
2. The radius is 10 in.
3. The diameter is 22 in.
4. The diameter is 28 in.
5. The radius is 13 in.
6. The diameter is 30 ft.
7. $R = 7$
8. $D = 18$
9. $R = 7.5$
10. $D = 17$
11. $R = 6\frac{1}{4}$
12. $D = 19$
13. $R = 5\frac{1}{3}$
14. $D = 21$

XII. SUPPLEMENTARY CONSTRUCTIONS BASED ON CIRCLES

78. Constructing the perpendicular bisector of a segment with compasses and ruler.

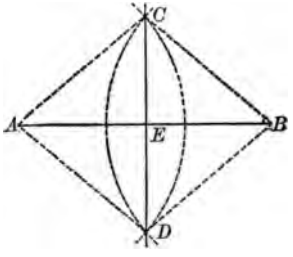
The **perpendicular bisector** of a segment is the perpendicular erected to the segment at its mid-point.

EXERCISE 101

1. *a.* Draw a segment AB.

b. With radius more than $\frac{1}{2}\ AB$, and center A, draw a small arc above the center of AB, and another below the center of AB.

c. With the same radius and B as center, draw small arcs cutting the two drawn previously, at points C and D.



d. Draw CD, intersecting AB at E.

e. Compare segments AE and EB. (See Ex. 2, p. 115.)

f. What kind of angles are $\angle\ AEC$ and $\angle\ CEB$? (Test them.)

g. Below, print: *CD is the perpendicular bisector of AB.*

2. Draw an oblique segment. Construct its perpendicular bisector.

3. Draw a vertical segment. Construct its perpendicular bisector.

4. Draw a horizontal segment. Divide it into four equal parts by means of three perpendicular bisectors.

5. *a.* Draw a reasonably large triangle. Construct the perpendicular bisector of each of its sides. What happens?

b. Compare the distances to the point of intersection from each of the three vertices.

6. Repeat Ex. 5, for a triangle of different shape. Do you obtain the same result?

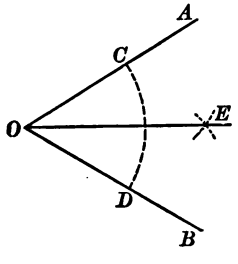
7. Draw any segment AB. Construct the perpendicular bisector of it. On this perpendicular bisector, take any point O. Draw OA and OB and compare them. Do the same for two more points on the perpendicular bisector. What seems to be true?

79. Constructing the bisector of an angle with compasses and ruler.

EXERCISE 102

1. *a.* Draw an acute angle AOB.

b. With any radius, and O as center, draw an arc cutting OA at C and OB at D.



c. With a radius more than one half CD, and C as center, draw an arc inside the angle, near where the bisector must pass. With the same radius, and D as center, draw a second arc cutting the first one at E.

d. Draw OE.

e. Compare $\angle AOE$ and $\angle EOB$. What name is given to the line that so divides an angle?

f. Below, print: *OE bisects $\angle AOB$.*

2. Draw an obtuse angle and bisect it.

3. Draw a second obtuse angle and divide it into four equal parts.

4. Draw a triangle of reasonably large size. Construct the bisectors of each of its angles. What happens?

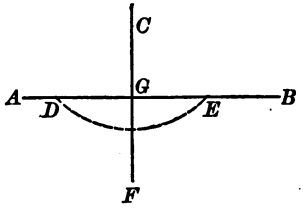
5. Repeat Example 4 for a triangle of different shape. What seems to be true?

80. Constructing the perpendicular to a line from a point not on the line by means of compasses and ruler.

EXERCISE 103

1. *a.* Draw any line AB, and place a point C, above AB.

b. With C as center and a radius long enough to cut AB, draw an arc cutting AB at D and at E.



c. With a radius more than one half DE, and D as center, draw an arc below the approximate center of DE. With the same radius, and E as center, draw a second arc, cutting the first one at F.

d. Draw CF, cutting AB at G.

e. Determine what kind of angles are formed at point G.

f. Below, print: *CF is perpendicular to AB from C.*

2. Repeat Example 1 for an oblique line.

3. Repeat Example 1 for a vertical line.

4. Draw a triangle of reasonably large size. From each vertex, construct the perpendicular to the opposite side, as in Examples 1 and 2. What happens?

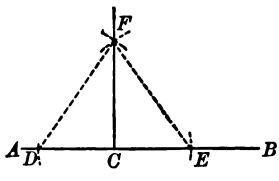
5. Repeat Example 4 for a triangle of different shape. What name is given to the line drawn from a vertex perpendicular to the opposite side?

81. Constructing the perpendicular to a line at a point on the line, by means of compasses and ruler.

EXERCISE 104

1. *a.* Draw any horizontal line AB, and place point C on it.

b. With any radius and C as center, draw two arcs, cutting AB at D and at E



c. With a radius more than one half DE, and D as center, draw a small arc over the approximate center of DE. With the same radius and E as center, draw a second arc, cutting the first one at F.

d. Draw CF. What kind of angles are formed at C? What kind of lines are AB and CF?

e. Below, print: *CF is perpendicular to AB at C.*

2. Repeat Example 1 for a vertical line.

3. Repeat Example 1 for an oblique line.

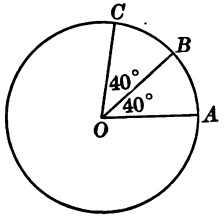
4. Draw a horizontal line. Place on it three points. At each, draw a perpendicular to the line. What kind of lines do you obtain?

Regular Polygons

***82. Dividing* a circle into equal arcs.**

EXERCISE 105

1. *a.* Draw a circle with radius $1\frac{1}{4}$ inches. Mark its center *O*. From *O*, draw two radii *OA* and *OB*, forming an angle of 40°; then draw radius *OC*, so that $\angle BOC$ shall also be 40°.



b. With tracing paper compare arc *AB* and arc *BC*.

c. Below, write: *$\angle AOB$ and $\angle BOC$ are central angles of the circle. When central angles are equal, they cut out from the circle* —— *arcs. Arc AB and arc BC each contain* 40 *arc degrees.*

2. *a.* Draw a circle with radius $1\frac{1}{4}$ inches. At its center, draw three central angles, each containing 60°. Compare their arcs.

b. In a second circle of the same size, see how many 60° angles can be drawn "side by side" around the center. Into how many equal parts do they divide the circle?

c. Draw the chord of each of the arcs in part *b*. Compare them.

d. Below, write: *If arcs of a circle are* ——, *the chords of these arcs also are* ——.

3. *a.* If you wish to divide a circle into four equal arcs, how many equal central angles must you have around the center? How large will each one be?

b. Draw a circle with radius 2 in. Using your protractor, make the equal angles which you need around the center. Test the arcs you obtain. If they are not equal, repeat the drawing, until they are approximately equal.

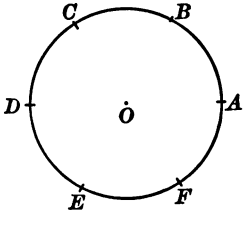
c. Draw the chords of these arcs. What kind of figure seems to be formed? Test its angles and sides.

d. Below, write: *A square inscribed in a circle.*

4. Make the same figure that was drawn for Example 3, without using your protractor. Draw a diameter of the circle. At the center, construct a second diameter perpendicular to the first. (See 81, p. 190.)

5. *a.* Divide a circle into six equal arcs, by using your protractor. Draw the chords of these arcs. Study the angles of this figure. Below, write: A **regular hexagon** inscribed in a circle. *It has six* —— *sides, and six* —— *angles.*

b. The same figure may be constructed without the protractor. Draw a circle with radius $1\frac{1}{4}$ in., center O. Draw radius OA. With the same radius, and A as center, draw an arc cutting the circle at B; with the same radius and B as center, draw a second arc, cutting the circle at C; and so on. If you are accurate, you will return to point A when you have done this six times. Compare these arcs by means of tracing paper. Draw the chords of the arcs, and measure the angles of the figure.



c. With center X draw a circle with radius $1\frac{1}{2}$ in. and construct a regular hexagon in it with your compass only.

d. Repeat part *c* in a circle with center Y and radius 1 in.

e. Repeat part *c* in a circle with center Z and radius $1\frac{3}{4}$ in.

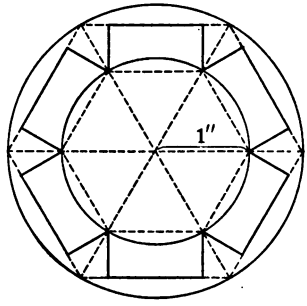
6. *a.* Draw a circle with radius $1\frac{1}{2}$ in. and, with your protractor, divide it into three equal parts. (How many equal central angles must you have? How large must each be?) Draw the chords of these arcs. Compare them and also the angles of the figure formed. What *seems to be* true?

b. Repeat part *a* in a circle of different size. Do you come to the same conclusion?

c. Below, write: **An equilateral triangle** inscribed in a circle. *It has three —— sides and three —— angles. Each angle measures —— degrees.*

7. The previous figure can be constructed without the protractor.

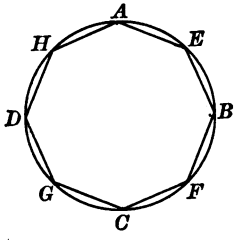
a. Draw a circle from center *O* having radius of $1\frac{1}{2}$ in. Divide it into six equal arcs as directed in Example 5, part *a*. Mark these points of division on the circle, in order, *A*, *B*, *C*, *D*, *E*, and *F*. Then draw segments *AC*, *CE*, and *EA*. Study the sides and angles of triangle *ACE*.



b. Repeat the previous drawing for a circle of different size.

8. Make a drawing of the pattern for a hexagonal box, pictured at the right. This is to scale. Make yours full size.

9. A regular octagon is a figure having eight equal sides and equal angles.



a. Draw a regular octagon, using your compasses, protractor, and ruler.

b. Draw a regular octagon, using only your compasses and ruler. You will need to use the construction of Example 4, p. 192, and of § 79, p. 188.

10. Some regular polygons cannot be constructed with compasses and ruler alone. Such polygons can be con-

structed approximately correct by using the protractor. One of them is the polygon having *seven sides*. Draw a regular polygon of seven sides, using your protractor, compasses, and ruler.

EXERCISE 106

Review of Circles

1. Draw a circle. Draw and mark a radius; a diameter; a chord; an arc.

2. Draw two concentric circles.

3. What is a semicircle?

4. What is the relation between a radius and a diameter of the same circle?

5. What is the length of the circle whose radius is 4.5 inches?

6. What is the length of the circle whose diameter is 7.5 inches?

7. What is another name for "length of a circle"?

8. Give the formula for the circumference of a circle.

9. What two numerical values of π are used in computation? Which is the more accurate?

10. How may the approximate value of π be determined?

11. Find by the formula the circumference when $R = 4.25$; also when $D = 6.7$.

12. What is the area of the circle whose diameter is 14 feet?

13. What is the area of the circle whose radius is 14 feet?

14. Give a rule for finding the area of a circle.

15. What does "area of a circle" mean?

16. How much is 8^2? 7^2? 11^2? 5.1^2?

17. Give the formula for the area of a circle.

18. Find A when $R = 6.8$.

19. Find A when $D = 8.4$.

20. Find C when $D = 11.5$.

EXERCISE 107

Review of Supplementary Constructions Based on Circles

1. *a.* Draw a vertical segment RS. Construct its perpendicular bisector.

b. Repeat, starting with an oblique segment.

2. *a.* Draw an obtuse angle. Construct its bisector.

b. Repeat, starting with an acute angle.

3. Draw a segment MN. Place a point O in it. Construct the perpendicular to it at the point O.

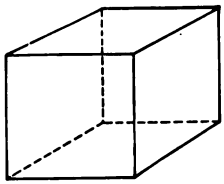
4. Draw a segment CD. Place a point not on it. Construct the perpendicular to CD from the point.

5. Draw a circle with radius $2\frac{1}{2}$ inches. By means of your protractor divide this circle into nine equal arcs. Draw the chords of these arcs. Compare them. Find the length of one of them to sixteenths of an inch.

6. In a circle with radius $2\frac{1}{2}$ inches, inscribe a regular hexagon, without using your protractor.

7. In a circle with radius $2\frac{1}{4}$ inches, inscribe a square, without using your protractor.

XIII. RECTANGULAR PARALLELOPIPED

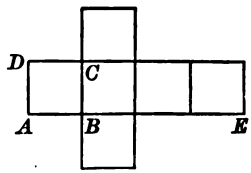


83. Cube. — The figure at the left represents a cube. There is probably a cube in your classroom, which you can examine.

EXERCISE 108

1. Make a list of all the facts which you observe about the faces, edges, and vertices of a cube. (Notice the kind of lines, the kind of angles, etc.)

2. You can make a cubical box out of cardboard or stiff paper by drawing a figure like the one below.



After making such a figure, cut it out from the cardboard along the outside lines. Then, on the face which you wish to have for the outside of your cube, cut part way through the cardboard along the inside lines of your figure, using the point of a penknife. Then turn the figure over on your desk, and bend each face upward, along the marks that show through the cardboard. The edges can be held together by gummed labels, or by gummed paper such as is used in stores in wrapping packages.

a. What kind of figure must $ABCD$ be? *b.* How long must AE be if the edges of the cube are to be 1 in. long?

NOTE. — Have most of the pupils make a 1 in. cube. Have one pupil make a 2 in. cube, and another a 3 in. cube, omitting the tops of these two. If this is inconvenient, obtain at least twenty-seven 1 in. cubes. With these, other figures can be formed.

A cube whose edge measures 1 in. incloses a cubic inch of space.

How many cubic inches of space are inside a cube one of whose edges is:

3. 2 in.?	5. 6 in.?	7. 8 in.?	9. 2.5 in.?
4. 3 in.?	6. 5 in.?	8. 10 in.?	10. 5.1 in.?

The number of cubic inches inside a cube is called the **volume of the cube** in cubic inches.

11. Make a rule for finding the volume of a cube.

12. What is the volume of a cube whose edge is 12 in.? What name is given to this amount of space?

13. A cubic foot of water weighs 62.5 lb. How much does a cubic yard of water weigh?

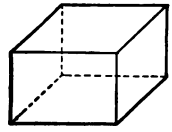
14. Clay soil is 2.65 times as heavy as water. What is the weight of a cubic yard of clay soil?

15. Soil rich in humus weighs about 2.5 times as much as water. What is the weight of a cubic foot of humus soil?

16. Iron weighs about 7.21 times as much as water. What is the weight of a cubic foot of iron?

17. Cork weighs about .24 times as much as water. What is the weight of a cubic foot of cork? What is the weight of a cubic inch of cork?

84. Rectangular parallelopiped.—The figure at the right represents a rectangular parallelopiped. The box in which crayon is usually supplied to schools is a familiar example of this solid.



NOTE. — This solid is often called a rectangular prism in elementary schools. That name is not sufficiently accurate, mathematically; *the one given in this text* is preferable.

EXERCISE 109

1. After examining carefully the figure in the book, and the crayon box, make a list of the facts you observe about the edges, faces, and angles on the faces of the rectangular parallelopiped.

a. Are the edges all equal, necessarily? How many different lengths of edges do you find? How many edges are there having any one length?

b. Are the faces necessarily square? May they be square?

2. Can you find in your classroom a rectangular parallelopiped?

3. The three edges meeting at any one vertex are called the **length**, **width**, and **height** of the rectangular parallelopiped.

Suppose a hollow rectangular parallelopiped is 3 in. wide, 4 in. long, and 2 in. high, inside measurements.

a. How many one-inch cubes could you place in the bottom of the box?

b. How many such layers could you place in the box?

c. How many cubic inches are there inside the box?

4. How many cubic inches are there inside a box if the width is 2 in., the length is 5 in., and the height is 3 in.?

5. Make a rule for finding the volume of a rectangular parallelopiped.

85. If w represents the number of linear units in the width, l the number of the *same kind of units* in the length, h *the number* in the height, and V the number of the *corresponding* cubic units in the volume, then the formula

for the volume of a rectangular parallelopiped is

$$V = lwh.$$

Remember that *lwh* means $l \times w \times h$.

Example. — If the length is 3 ft. 6 in., the height is 2 ft. 4 in., and the width is 12 ft., find the volume.

Solution. — 1. Since $V = l \times w \times h$

2. then $V = 3\frac{1}{2} \times 2\frac{1}{3} \times 12$

3. $= \frac{7}{2} \times \frac{7}{3} \times 12 = 98$ cu. ft.

EXERCISE 110

Find, as above, the volume V, when:

1. $l = 6$ ft.; $w = 3$ ft.; and $h = 4$ ft.

2. $l = 8$ yd.; $w = 7$ yd.; and $h = 2$ yd.

3. $h = 5$ ft. 6 in.; $l = 16$ ft. 6 in.; and $w = 8$ ft.

4. $w = 3$ ft. 4 in.; $h = 1$ ft. 3 in.; and $l = 6$ ft. 8 in.

5. $l = 10$ yd. 1 ft.; $h = 2$ yd.; and $w = 6$ yd. 2 ft.

6. How many cubic feet of material must be moved in digging a cellar 24 ft. wide, 28 ft. long, and 6 ft. deep?

7. What is the cost of excavating the cellar in Example 6 at 75¢ per cubic yard?

8. A tank for holding water is 2 ft. high, 2 ft. 6 in. wide, and 4 ft. long.

a. How many cubic feet of water will it hold when full?

b. Since there are about $7\frac{1}{2}$ gal. of water in one cubic foot, how many gallons of water will the tank hold?

c. If one cubic foot of water weighs $62\frac{1}{2}$ lb., what is the *weight of the water* in the tank, when the tank is full?

9. How many cubic feet of concrete are needed for a sidewalk 40 ft. long, 5 ft. wide, and 4 in. thick?

10. How many cubic yards of concrete are needed for 50 ft. of concrete road which is 16 ft. wide and averages 6 in. in thickness?

11. If a ton of soft coal occupies about 38 cu. ft., how many tons of coal will an open freight car hold, which is 36 ft. long, 8 ft. 4 in. wide, and 3 ft. high?

12. How many tons of soft coal can be stored in a coal bin 14 ft. long and 5 ft. wide, if the coal can be piled 6 ft. high?

13. A ton of timothy hay occupies about 500 cu. ft. How many tons, approximately, are there in a stack which is 30 ft. long, 15 ft. wide, and 12 ft. high?

14. A bushel of small grain (oats, wheat, etc.) occupies about $1\frac{1}{4}$ cu. ft. How many bushels of oats can be stored in a bin which is 10 ft. long, 8 ft. wide, and 5 ft. high?

15. A farmer needs a bin to hold about 500 bu. of oats. He can make it 9 ft. wide and 12 ft. long. How high must he make it?

16. A bushel of potatoes occupies about $1\frac{1}{2}$ cu. ft.

a. How many bushels can be stored in a bin 20 ft. long, 6 ft. wide, and 5 ft. high?

b. How high must a bin be to hold 20 bu. if it is 2 ft. 6 in. wide and 5 ft. long?

17. A schoolroom is 25 ft. by 33 ft. by 12 ft. How many cubic feet of air are there in the room?

18. A cord of wood is 4 ft. wide, 4 ft. high, and 8 ft. long. How many cubic feet are there in a cord?

19. About how many cords of wood are there in a pile *which is 8 ft.* wide, 6 ft. high, and 120 ft. long?

20. Cast iron weighs about .26 lb. per cu. in. What is the weight of a cast iron bar which is 2 in. wide, $1\frac{1}{2}$ in. thick, and 5 ft. long?

21. A cubic foot of ice weighs about 58 lb. About how much does a piece weigh which is 14 in. thick, 22 in. long, and 15 in. wide?

22. How many gallons of water does it take to fill a high school swimming pool which is 20 ft. wide, 35 ft. long, and averages 5 ft. in depth? (See Example 8.)

23. A coal dealer has a bin of soft coal which is 20 ft. wide, 25 ft. long, and averages 10 ft. in height. About how many tons of coal does it contain?

24. A road contractor has to remove soil to an average depth of $1\frac{1}{2}$ ft. on a street 40 ft. wide, for a distance of $\frac{1}{4}$ mile. (1 m. = 5280 ft.)

a. How many cubic yards of soil have to be removed?

b. If his teams can haul away 1 cu. yd. in each load, and if each team can haul 6 loads per day, how many days will it take 15 teams to remove the soil?

c. What will be the contractor's expense for teaming if he pays \$8.50 per day for the teams?

25. Make up an example like one of the Examples 6 to 24 of this list; or, better still, measure some coal bin, grain bin, piece of ice, tank, or other object having the form of a rectangular parallelopiped, and make up the problem suggested by the object.

EXERCISE 111

Miscellaneous Review Problems

1. Find the cost of 150 barrels of salt pork at \$18.757 per barrel.

2. *Change* $\frac{1}{8}$ *to* a decimal, carried out to three places.

3. Change $\frac{9}{16}$ to a three-place decimal.

4. Certain boards are .875 in. thick. How high will a pile of these boards be if there are 48 boards in the pile?

5. In 1919, about 28,199,000 bu. of potatoes were harvested in Wisconsin. *a.* About 39% of these were shipped to Chicago. How many bushels were shipped to Chicago? *b.* Only about 3% of the potatoes raised in Wisconsin were used inside the state. How many bushels were used within the state?

6. Find the semi-annual interest and the amount due on $950 at 5%.

7. Find the quarterly interest and the amount due on $750 at 6%.

8. Farmers who were expecting a crop of 45 bu. of wheat per acre early in 1919 got, in many cases, only 15 bu. per acre, because of the unfavorable weather. What per cent of the expected crop did they receive?

9. An attorney received 15% commission for handling a suit against a street car company for damages for injuries received by a client. If the court awarded the client $750, how much commission should the attorney receive?

10. A farmer received for six cows $185, $216, $320, $400, $275, and $315 respectively. What was the average price received?

11. A man borrowed $600 at his bank for 75 days, agreeing to pay 6% interest. *a.* How much interest must he pay when the money falls due? *b.* He invested the money in such manner that he would receive a return of $15 on the money during the 75 days. How much does *he make by the* transaction?

12. A coat had been marked $75. It was sold for $62.50. What per cent of the original price was the discount?

13. Find the base when the rate is 7% and the percentage is 217.

14. In a revised list of prices issued in October, 1920, one dealer printed:

Article	Old Price	New Price
Work shirt	$ 2.00	$ 1.45
Man's hat	$ 7.50	$ 6.50
Suit of overalls	$ 4.50	$ 3.50
10 yd. white muslin	$ 4.28	$ 2.65
Misses' coat	$24.95	$19.95

What per cent of the old price is the reduction in each case?

15. Find the semi-annual interest on $550 worth of Liberty Bonds paying 4½% interest.

16. What is the sale price of an article listed at $18.00 with a discount of 15%?

17. Gold weighs 19.26 times as much as water. One cubic foot of water weighs 62.5 lb. How much would a cubic foot of gold weigh?

18. A retail merchant bought from a wholesale dealer an article listed at $18.50, less a discount of 15%.

a. What did he pay for the article?

b. What must he receive for it in order to make a profit of 15% of his purchase price?

c. Later on, he sells it to a favored customer at a discount of 5% from his own list price. What does the purchaser pay?

d. What per cent of the cost to the merchant does the merchant make?

19. A merchant bought an article for 2¢ and sold it for 10¢.

a. What per cent of the cost was the profit?

b. What per cent of the selling price was the profit?

20. Find the interest on \$1250 at $5\frac{1}{2}\%$, borrowed on March 12 and repaid on May 29. Count the actual number of days.

21. A man, greatly in need of money, applied to a money lender for a loan of \$150 for 30 days. The money lender charged him a fee of \$5 for making the loan, and also interest at the rate of 8% per year.

a. How much interest does he have to pay for the 30 days?

b. What is the total amount received by the money lender for the use of his money for the 30 days? How much would this be per year?

c. What per cent of the principal, \$150, is this amount?

d. If the man had borrowed the money at a bank charging 6% interest, what would have been the total cost for the use of the money for the 30 days?

22. In 1910, the population of the United States was 92,000,000. In 1920, it was 105,000,000.

a. What per cent of the population in 1910 was the increase in population by 1920?

b. If the population increases at the same rate during the next decade, what will the population be in 1930?

23. *a.* Write a promissory note of which Henry Harris is the maker and Charles Adams is the payee, for \$2500, with interest at 6%, payable at some bank in your town *in 90 days.*

b. Find the amount due on the note on the date of maturity.

24. A furniture dealer agreed to supply all the furniture and rugs for a new home at wholesale price plus a commission of 10% of that price.

a. The wholesale cost of the goods purchased was $3275. What was the commission of the dealer, and what was the total cost to the purchaser?

b. If merchants in that town marked their goods, on the average, at an advance of 25% over the wholesale price, how much would these goods have cost the purchaser if he had bought them at various stores, out of stock?

c. Tell some reasons why the furniture dealer could afford to sell the goods for the commission of 10%.

25. Of what number is 640: *a.* 5%? *b.* 4%? *c.* $12\frac{1}{2}$%? *d.* $16\frac{2}{3}$%? *e.* $33\frac{1}{3}$%? *f.* $37\frac{1}{2}$%? *g.* 20%? *h.* $62\frac{1}{2}$%? *i.* $83\frac{1}{3}$%? *j.* $87\frac{1}{2}$%? *k.* $8\frac{1}{3}$%?

26. *a.* In October, 1920, an automobile that had been selling for $5400 was reduced to $4500. What per cent of the former price was the reduction?

b. If the agent received $12\frac{1}{2}$% of the selling price of the car, how much less was his income per car?

27. Problem. A study of the records of an arithmetic class. 40 pupils entered the class.

a. 6 were repeatedly absent. What per cent of the class were they?

b. 4 pupils finally dropped out during the year. What per cent of the class were they?

c. Of the pupils who remained, 3 failed to pass, 9 received the grade poor, 15 the grade fair, 6 the grade good,

and 3 the grade excellent. What per cent of the ones remaining received each grade?

d. Draw a line graph representing each of the groups of pupils.

28. Problem. A study of the profits on the sale of an article.

a. An article was bought for $1.60. What must be charged for it to make a gross profit of 25% of the cost?

b. What per cent of the selling price is the gross profit?

c. If the expense involved in selling the article is 10% of the selling price, what was the net profit?

d. What per cent of the cost was the net profit?

e. What per cent of the selling price was the net profit?

29. An article cost $3.60 and sold for $4.20.

a. What per cent of the cost was the gross gain?

b. If the expense involved in selling the article was 10% of the selling price, what was the net profit, and what per cent of the cost was it?

30. Find the interest on $625 borrowed on December 12, 1920, and repaid on February 27, 1921, if the rate of interest is 8%. Count months and days.

31. An article was listed at $85.

a. It was offered for sale at a discount of 20%. What was the sale price?

b. It cost the merchant $60. What was his gross profit on the sale?

c. If the actual expense in buying and selling the article had been 5% of the cost price to the merchant, did the merchant actually make a net profit and how much, when *he sold it at the* discount price?

32. A commission merchant received 30,000 lb. of potatoes from a farmer. He sold them for \$1.75 per cwt. He charged the farmer a commission of 5% of the sale price. How much should he send the farmer?

33. A man whose rent was \$50 per month in a heated apartment was notified that the rent was increased to \$75 per month.

a. What per cent of the former rent was the increase?

b. The man's salary had been \$175 per month. What per cent of his annual salary was his annual rent at the old rate?

c. About the time when his rent was increased, his salary also was increased to \$200 per month. What per cent of his new annual salary was his new annual rent?

34. A clerk in a dry goods store received \$20 per week pay, and a commission of 3% on all sales over \$100 per week. If her sales amounted one week to \$285, what was her total income for the week?

35. The interest on certain Liberty Bonds is payable on October 15 and April 15.

a. How much interest should a man receive on October 15 on \$7500 worth of bonds, if the rate is $4\frac{1}{4}\%$?

b. On December 15th, he decides to sell the bonds. He is offered 91.25% of the face value of the bonds, and interest on the bonds from October 15th to date. How much should he receive?

36. Sugar once sold for 6¢ per pound. In the summer of 1920, it sold for 36¢ per pound. What per cent of the former price was the increase?

37. Problem. Study the rate of profits on various *sales of a wholesale* purchase of apples.

a. A grocer bought 3300 lb. of apples at 6¢ per pound. What was the total cost?

b. The commission merchant made a profit of 10% of the price at which he sold them. What were his profits?

c. The grocer sold about 10% of the apples at 10¢ per pound. What per cent of the cost was his profit on such sales? What were his gross profits on these sales?

d. He sold about 50% of the apples at 3 lb. for 25¢. What per cent of the cost was his profit on such sales? What were his gross profits on these sales?

e. He sold the balance at $3.25 per bushel. What per cent of the cost was his profit on such sales, if a bushel contains 44 lb.? What were his gross profits on such sales?

f. What was the total of his gross profits in handling the apples? What per cent of the cost is this gross profit?

38. Problem. To estimate the cost and a fair selling price of some pot-grown tulips.

a. A florist bought 1000 tulips for $30.00, less 5% for cash payment. What was the net cost of the bulbs?

b. He decided to place them in 5 in. bulb pots, five bulbs in a pot. How many pots will he need?

c. What will the pots cost him, at the rate of $37.50 per thousand, less a discount of 10%?

d. He estimates that one of his workmen will fill 40 pots in one hour. If the man receives $4.50 per 8 hr. day, what will be the cost of placing the bulbs in the pots?

e. He estimates that it will take 2 hours for a man to place the pots in their place for winter storage, and 4 hours more to dig them out again in the spring and get them ready for sale. What is this additional cost for labor?

f. *What is* the total cost of all these items?

g. In order to pay all other expenses of his business and make what he considers a reasonable net profit, he decides to add to the cost, as found in part *f*, 150% of itself. What does this make for the selling price of the whole lot of tulips? What is the selling price per pot?

39. Problem. To estimate the cost of materials for draperies for 5 semicircular windows, each of which is 4 ft. in diameter.

a. Cretonne is obtainable with a pattern such that the "up and down" dimension of the drapery for each window could be cut from the width of the material by cutting the "left to right" dimension of the drapery from the length of the material. How many yards of the material have to be purchased, and what is the cost at $2.50 per yard?

b. How many yards of edging are required for the semicircular edges of the draperies? What is the cost at 12¢ per yard?

c. How many yards of fringe are required for the straight edges? What is the cost at 18¢ per yard?

d. What is the total cost for the materials?

40. Tell how to find the average of several numbers.

41. *a.* What is the formula for the *percentage* when the *base* and the *rate* are known?

b. What is the rule?

c. Bring in an example in which you can apply this rule.

42. *a.* What is the formula for finding the *rate* when the *percentage* and the *base* are known?

b. What is the rule?

c. Bring in an example in which you can apply this *rule.*

43. *a.* What is the formula for finding the *base* when the *percentage* and the *rate* are known?

b. What is the rule?

c. Bring in an example in which you can apply this rule.

44. *a.* What is the difference in meaning between " gross " profit and " net " profit?

b. Tell how to determine the selling price when the cost and the rate of gross profit on the cost are known.

c. Bring in an example that will make clear the meaning of these words.

45. *a.* What is a " discount "?

b. Tell some reasons why a discount is given.

c. Tell how to find the selling price when the list price and the rate of discount are known.

d. Bring in an example involving a discount.

46. *a.* What is meant by a " commission "?

b. Do you know any persons who are paid on a commission basis?

c. What reasons can you think of for paying persons in this manner?

d. If possible, bring in an example of some person who was paid a commission for his services.

47. *a.* What is meant by " interest " on money?

b. Why is interest paid?

c. What is meant by the " amount "?

48. *a.* What is a promissory note?

b. Who is the " maker " of the note?

c. Who is the " payee " of the note?

d. *Write* a promissory note having the proper form.

SUPPLEMENTARY ABSTRACT DRILL EXERCISES

The following exercises should be used according to the teacher's judgment of the needs of the class and of individual pupils of the class. Many teachers spend a few minutes (*e.g.* five minutes) daily (or frequently) on such drill work for the whole class, and then assign special practice to pupils who are found to be slow or inaccurate.

For convenience in making assignments, the exercises are grouped according to the various processes. If a miscellaneous exercise is wanted, select one part from each of a number of exercises; *e.g.* one each from Exercises 113, 114, 115, 116, etc.

EXERCISE 112

Two Column Addition Drill

DRILL TABLE

	I	II	III	IV	V	VI	VII	VIII	IX	X
I	13	38	27	44	78	35	39	86	64	57
II	59	17	48	23	45	29	16	94	76	32
III	18	33	49	15	26	24	52	37	89	78
IV	47	19	54	32	96	43	25	28	66	85
V	95	62	53	36	34	67	68	12	45	79
VI	74	85	46	57	63	58	79	92	36	24
VII	66	64	82	69	77	83	73	55	48	95
VIII	73	56	65	72	84	99	87	83	29	36
IX	58	72	39	43	99	25	74	46	63	37
X	26	94	87	35	52	48	62	39	76	24

SUGGESTED EXERCISES

(Do not do two consecutive ones on the same day.)

1. Hold a slip of paper below Row II. Write upon it the sum of the two numbers in each column that appear above *the paper.* (Ten sums.)

2. Hold a slip of paper below Row III. Write upon it the sum of the three numbers in each column that appear above the paper.

3–8. Similarly obtain the sum of the numbers in each column that are in first 4, 5, . . . 9 rows.

9. (Speed and accuracy test.) Hold a slip of paper below Row X. Upon it write the sum of the numbers in each column that appear above the paper.

NOTE 1. — **Median time.** Direct the pupils to raise their hands when they finish Example 9. Note the number of minutes and seconds that elapse until one half the class is through; *that is the median time for the example.* Record this. It is a means of comparing the ability, with respect to this example, of this class with any other class; also, any pupil who requires more than the median time for this example is below the median in speed, and should practice until his speed is increased.

NOTE 2. — **Median accuracy.** After all have finished, read the correct results. Tell the pupils to check their correct results and to cross out the incorrect ones. Tell them to write below their names the percentage of correct results.

Direct all who made 100, 90, 80, etc., to stand until one half the class are standing. The mark of the last pupil to rise is the median accuracy mark of the class for this example. Record this and use it as a means of comparing this class with other classes, and of selecting pupils who need further drill.

If time cannot be allowed for all the pupils to finish this or similar exercises, arbitrarily stop the class when one half are through, or any other part. In this case omit the computation of the "median accuracy" as described in this paragraph, and use the median time as a means of selecting pupils who need special drill.

NOTE 3. — Further exercises can be made by obtaining the sums of the numbers in the *lower* two rows, or three rows, etc.

EXERCISE 113

Drill in Adding Three or More Columns

DRILL TABLE

	I	II	III	IV	V	VI	VII	VIII
I	232	764	673	569	67.1	69.3	15.762	204.35
II	425	836	870	749	298.	2.65	5.47	62.54
III	357	272	342	646	580.	53.4	63.25	982.47
IV	635	380	456	375	39.6	72.9	4.827	7.52
V	284	594	237	865	65.3	4.56	5.634	763.09
VI	473	809	862	467	707.	851.	2.85	76.89
VII	898	769	959	433	560.7	5.78	3.057	19.26
VIII	469	386	489	268	83.	38.42	28.43	547.84
IX	371	563	225	752	73.8	941.20	71.85	675.92
X	485	627	780	924	49.6	356.28	18.035	87.63

SUGGESTED EXERCISES

See the exercises suggested in connection with Exercise 112.

For other addition exercises, use the Drill Tables provided for Exercises 114 and 115, which follow.

EXERCISE 114

Written Subtraction Drill

DRILL TABLE

	I	II	III	IV	V	VI
I	83745	59290	76305	537.832	69.458	295.659
II	81629	52947	75286	462.921	58.4	265.806
III	78564	41392	63924	308.52	51.025	197.083
IV	63946	30685	47692	285.061	39.843	164.096
V	51809	19890	32574	131.5	26.587	104.009
VI	30543	11508	23064	97.62	19.07	95.893
VII	21845	6295	18547	73.007	8.026	78.89
VIII	19854	1463	12468	17.295	3.272	49.999
IX	9573	1258	6752	15.096	2.567	35.876
X	3589	264	697	5.984	2.541	27.082

SUGGESTED EXERCISES

1. Place a strip of paper below Row X. Upon it write the remainders when you subtract the numbers in Row X from the ones above in Row IX.

2–9. Similarly subtract the numbers of Row IX from those of Row VIII; those of Row VIII from those of Row VII; etc.

10. Subtract the numbers of Row III from those of Row I.

11. Subtract the numbers of Row VIII from those of Row IV, without copying the numbers from the book, etc.

12. Subtract the numbers of any selected row from those of any selected row preceding.

EXERCISE 115

Multiplication Drill

DRILL TABLE

	I	II	III	IV	V	VI	VII
I	15	21	43	2.6	.36	.286	48.251
II	28	48	79	.85	52.1	3.25	67.842
III	209	576	857	62.4	44.37	47.225	89.541
IV	1275	3184	5317	432.9	752.8	563.97	76.239
V	4832	5763	6234	762.6	86.59	31.416	86.517
VI	5547	6832	7895	827.09	741.82	654.517	87.348
VII	6619	7091	8060	945.07	872.15	763.974	16.008
VIII	7995	8763	9841	35.06	36.78	59.84	98.975

SUGGESTED EXERCISES

1. Multiply each number of Row I:

a. by 3; *b.* by 5; *c.* by 6; *d.* by 7; *e.* by .08; *f.* by .9.

Use the other rows in similar manner, if necessary.

2. Multiply the numbers of any row by 10, or 100, or .1, or .001, etc.

3. Multiply the numbers of Row I by 45.

4. Multiply the numbers of Row II by 57.

5. Multiply the numbers of Row III by 36.2.

6. Multiply the numbers of Row IV by 78.

7. Multiply the numbers of Row V by 94.6.

8. (Speed and accuracy test.) Multiply the numbers in Row VI by 37. Obtain and record the medians.

9. (Speed and accuracy test.) Multiply the numbers in Row VII by 4.9. Obtain and record the medians.

10. (Speed and accuracy test.) Multiply the numbers in Row VIII by 6.8. Obtain and record the medians. Assign special drill for pupils who need it.

EXERCISE 116

Division Drill

Use the drill table of Exercise 115

SUGGESTED EXERCISES

Divide to two decimal places the numbers:

1. In Row I, *a.* by 5; *b.* by 6; *c.* by .8; *d.* by 7; *e.* by .9.

If necessary, use the other rows in like manner.

2. In Row II by 23.

3. In Row III by 37.5.

4. In Row IV by 58.4.

5. In Row V by 26.48.

6. (Speed and accuracy test.) Divide to two decimal places the numbers in Row VI by 75. Obtain and record *the medians.*

7. (Speed and accuracy test.) Divide the numbers in Row VII by 68.

8. (Speed and accuracy test.) Divide the numbers in Row VIII by 49. Assign special drill for pupils who need it.

EXERCISE 117

Drill on Fractions

In each of the following examples, *a.* add the two fractions or mixed numbers; *b.* subtract the smaller fraction from the larger; *c.* multiply the two fractions; and *d.* divide the first fraction by the second.

1. $\frac{1}{2}$; $\frac{1}{3}$
2. $\frac{3}{4}$; $\frac{1}{6}$
3. $\frac{3}{4}$; $\frac{3}{5}$
4. $\frac{3}{5}$; $\frac{5}{8}$
5. $\frac{2}{3}$; $\frac{5}{6}$
6. $\frac{3}{8}$; $\frac{2}{3}$
7. $\frac{7}{8}$; $\frac{3}{4}$
8. $\frac{5}{6}$; $\frac{3}{8}$
9. $\frac{3}{5}$; $\frac{5}{6}$
10. $\frac{2}{3}$; $\frac{7}{8}$
11. $\frac{5}{16}$; $\frac{3}{4}$
12. $\frac{13}{16}$; $\frac{5}{8}$
13. $\frac{7}{20}$; $\frac{5}{12}$
14. $\frac{9}{16}$; $\frac{4}{3}$
15. $\frac{7}{32}$; $\frac{3}{2}$
16. $6\frac{1}{2}$; $5\frac{2}{3}$
17. $11\frac{1}{4}$; $6\frac{5}{8}$
18. $12\frac{1}{2}$; $18\frac{1}{3}$
19. $37\frac{1}{2}$; $87\frac{1}{2}$
20. $66\frac{2}{3}$; $33\frac{1}{3}$
21. $83\frac{1}{3}$; $8\frac{1}{3}$
22. $62\frac{1}{2}$; $37\frac{1}{2}$
23. $3\frac{3}{4}$; $11\frac{1}{5}$
24. $9\frac{5}{8}$; $6\frac{2}{3}$
25. $16\frac{1}{2}$; $5\frac{1}{6}$
26. $5\frac{11}{16}$; $8\frac{9}{16}$
27. $41\frac{3}{4}$; $6\frac{2}{3}$
28. $15\frac{3}{4}$; $5\frac{1}{2}$
29. $31\frac{1}{4}$; $12\frac{1}{2}$
30. $256\frac{1}{4}$; $8\frac{1}{3}$

EXERCISE 118

Fractional Parts

What is:

1. $\frac{5}{8}$ of 96?
2. $\frac{3}{16}$ of 256?
3. $\frac{6}{5}$ of 45?
4. $\frac{3}{4}$ of 60?
5. $\frac{11}{12}$ of 108?
6. $\frac{5}{11}$ of 121?
7. $\frac{3}{40}$ of 200?
8. $\frac{5}{12}$ of 90?
9. $\frac{9}{16}$ of 144?
10. $\frac{13}{5}$ of 300?
11. $\frac{3}{20}$ of 640?
12. $\frac{9}{25}$ of 8.25?
13. $\frac{5}{32}$ of 1.92?
14. $\frac{7}{12}$ of 1440?
15. $\frac{8}{15}$ of 465?
16. $\frac{5}{16}$ of 19.32?
17. $\frac{3}{8}$ of 42.48?
18. $\frac{8}{11}$ of 297?
19. $\frac{7}{16}$ of 3.84?
20. $\frac{4}{15}$ of 315?
21. $\frac{1}{8}$ of 6.27?

Of what number is:

22. 256 five sixths?
23. 45 nine sixteenths?
24. 60 two thirds?
25. 108 nine eighths?
26. 121 eleven halves?
27. 200 ten thirds?
28. 90 fifteen sixteenths?
29. 144 nine sixteenths?
30. 300 fifteen thirty-seconds?
31. 640 five eighths?
32. 8.25 three twentieths?
33. 1.92 twelve fifths?
34. 1440 nine sixteenths?
35. 465 fifteen thirty-seconds?
36. 19.32 eight thirds?
37. 42.48 four fifths?
38. 297 eleven thirds?
39. 315 five fourths?
40. 3.84 sixteen ninths?

What part is:

41. 16 of 96?
42. 72 of 256?
43. 25 of 45?
44. 42 of 60?
45. 18 of 108?
46. 33 of 121?
47. *65 of 200?*
48. 27 of 90?
49. 48 of 144?
50. 75 of 300?
51. 25 of 640?
52. 8.25 of 33?
53. 1.92 of 5.12?
54. 440 of 1440?
55. 465 of 775?
56. 19.32 of 25.76?
57. 1.416 of 42.48?
58. 297 of 1039.5?
59. .32 of 3.84?
60. 315 of 504?
61. 471 of 785?

NOTE. — If speed and accuracy tests on the use of fractional parts are desired, Examples 11–20 can be used for one test, Examples 31–40 for the second test, and Examples 41–60 for the third. In each case, obtain and record for future use the median time and the median accuracy for each test.

If miscellaneous arrangement of the three kinds of examples is wanted, and that in most cases will prove desirable, do, on one day, Examples 1, 21, and 41; on another day, Examples 2, 22, and 42.

EXERCISE 119

Percentage Drills

What is:

1. 60% of 660?
2. $12\frac{1}{2}$% of 475?
3. $16\frac{2}{3}$% of 39?
4. $8\frac{1}{2}$% of 243?
5. 150% of 58?
6. $8\frac{1}{3}$% of 84?
7. 120% of 35?
8. $83\frac{1}{3}$% of 42?
9. 11% of 113?
10. $37\frac{1}{2}$% of 76?
11. $5\frac{1}{2}$% of 27.5?
12. $66\frac{2}{3}$% of 963?
13. 40% of 5780?
14. 18% of 36.5?
15. 31% of 84.7?
16. 65% of 200?
17. 23% of 53.7?
18. $62\frac{1}{2}$% of 69.4?
19. $11\frac{1}{9}$% of 8253?
20. $3\frac{3}{4}$% of 116.5?
21. $4\frac{1}{4}$% of \$1500?
22. 95 is 5% of what number?
23. 13 is 6% of what number?
24. 72.9 is 9% of what number?
25. 2.9 is $8\frac{1}{3}$% of what number?
26. 126 is 14% of what number?
27. 77 is 25% of what number?
28. 147 is $33\frac{1}{3}$% of what number?
29. 9.04 is 40% of what number?
30. 8.36 is 75% of what number?
31. 46 is 23% of what number?
32. 205 is $2\frac{1}{2}$% of what number?
33. *32.8* is 4% of what number?

34. 57.6 is $16\frac{2}{3}\%$ of what number?
35. 915 is 15% of what number?
36. 248 is 75% of what number?
37. 63.5 is $83\frac{1}{3}\%$ of what number?
38. 14.2 is 5% of what number?
39. 5.92 is $3\frac{1}{2}\%$ of what number?
40. 83.7 is 7% of what number?

What per cent is:

41. 110 of 660?
42. 95 of 475?
43. 13 of 39?
44. 72.9 of 243?
45. 2.9 of 58?
46. 126 of 84?
47. 77 of 35?
48. 147 of 42?
49. 9.04 of 113?
50. 8.36 of 76?
51. 46 of 230?
52. 205 of 100?
53. 32.8 of 40?
54. 18 of 540?
55. 915 of 505?
56. 62 of 248?
57. 63.25 of 23?
58. 14.2 of 69.4?
59. 1376 of 8256?
60. 83.7 of 111.6?
61. 111.25 of 133.5?

NOTE. — See the note following Exercise 118.

TABLES OF MEASURES

Measures of Length or Linear Measure

12 inches (in.)	= 1 foot (ft.)
3 feet	= 1 yard (yd.)
$5\frac{1}{2}$ yards or $16\frac{1}{2}$ feet	= 1 rod (rd.)
320 rods 1760 yards 5280 feet	= 1 mile (mi.)
1 knot or nautical mile	= 6080 ft. (U. S.)

Surveyors, in measuring land, use a chain (ch.) the length of which is 4 rods, divided into 100 links (li.) of 7.92 inches each. 80 chains = 1 mile.

Measures of Surface or Square Measure

144 square inches (sq. in.)	= 1 square foot (sq. ft.)
9 square feet	= 1 square yard (sq. yd.)
$30\frac{1}{4}$ square yards	= 1 square rod (sq. rd.)
160 square rods 43560 square feet	= 1 acre (A.)
640 acres	= 1 square mile (sq. mi.)
10 square chains	= 1 acre
36 square miles	= 1 township

Measures of Volume or Cubic Measure

1728 cubic inches (cu. in.)	= 1 cubic foot (cu. ft.)
27 cubic feet	= 1 cubic yard (cu. yd.)
128 cubic feet (a pile 4 ft. high, 4 ft. wide, and 8 ft. long)	= 1 cord

Liquid Measures

4 gills (gi.)	= 1 pint (pt.)
2 pints	= 1 quart (qt.)
4 quarts	= 1 gallon (gal.)

It has been customary to consider $31\frac{1}{2}$ gallons the contents of a *barrel (bbl.)* and 63 gallons the contents of a hogshead. In practice,

however, barrels are of various sizes; thus the standard gasoline barrel contains 56 gallons. A liquid quart contains 57¾ cubic inches; a liquid gallon 231 cubic inches.

Dry Measures

(Used in measuring grain, vegetables, fruits, etc.)

2 pints (pt.)	= 1 quart	(qt.)
8 quarts	= 1 peck	(pk.)
4 pecks	= 1 bushel	(bu.)

A dry quart contains 67⅕ cubic inches; a bushel contains 2150.42 cubic inches.

Avoirdupois Weight

16 ounces (oz.)	= 1 pound	(lb.)
100 pounds	= 1 hundredweight	(cwt.)
20 hundredweight 2000 pounds	= 1 ton	(T.)

At United States Custom Offices, and at iron and coal mines, the following weights are used:

112 pounds = 1 long hundredweight
2240 pounds = 1 long ton

Troy Weight

(Used in weighing gold, silver, and jewels)

24 grains (gr.)	= 1 pennyweight	(pwt.)
20 pennyweights	= 1 ounce	(oz.)
12 ounces	= 1 pound	(lb.)

Apothecaries' Weight

20 grains (gr.)	= 1 scruple	(℈)
3 scruples	= 1 dram	(ʒ)
8 drams	= 1 ounce	(ʒ)
12 ounces	= 1 pound	(℔)

Apothecaries' Liquid Measures

60 minims (m.)	= 1 fluid dram	(f ʒ)
8 fluid drams	= 1 fluid ounce	(f ʒ)
16 fluid ounces	= 1 pint	(℗)

Measures of Time

60 seconds (sec.)	= **1 minute**	(min.)
60 minutes	= **1 hour**	(hr.)
24 hours	= **1 day**	(da.)
7 days	= **1 week**	(wk.)
52 weeks / **365 days**	= **1 year**	(yr.)
366 days	= **1 leap year**	

If the number of a year is divisible by 4 but not by 100, the year is a leap year; also when the number of the year is divisible by 400 the year is a leap year. In leap years, February has 29 days.

The Solar Year has nearly $365\frac{1}{4}$ days.

Angle and Arc Measures

60 seconds ($''$)	= **1 minute**	($'$)
60 minutes	= **1 degree**	($^\circ$)

A quadrant contains 90 arc degrees: a circle contains 360 arc degrees. A right angle contains 90 angular degrees.

The length of a degree ($\frac{1}{360}$) of the earth's equator is about $69\frac{1}{8}$ miles.

English Money

4 farthings (far.)	= **1 penny**	(d.)
12 pence	= **1 shilling**	(sh.)
20 shillings	= **1 pound or sovereign**	(£)
	= **$4.8665**	

French Money

10 centimes (ct.)	= **1 decime**	(dc.)
10 decimes	= **1 franc**	(fr.)

1 franc = 19.3 cents

German Money

100 pfennige = 1 mark

1 mark = 23.8 cents

Miscellaneous Measures

Numbers	Paper
12 units = 1 dozen (doz.)	**24 sheets = 1 quire**
20 units = 1 score	**20 quires = 1 ream**
12 dozen = 1 gross (gr.)	**2 reams = 1 bundle**
***12 gross* = *1* great gross**	**5 bundles = 1 bale**

Some Standard Weights

(Used in most of the States)

Wheat	60 lb. = 1 bushel
Rye	56 lb. = 1 bushel
Oats	32 lb. = 1 bushel
Barley	48 lb. = 1 bushel
Corn, on cob	70 lb. = 1 bushel
Corn, shelled	56 lb. = 1 bushel
Potatoes	60 lb. = 1 bushel
Water	1 cu. ft. = $62\frac{1}{2}$ lb.

Carat. (1) A term used to express the purity of gold. Pure gold = 24 carats fine. If 2, 4, etc., parts of alloy are present, the gold is 22, 20, etc., carats fine. Thus, 14 carat gold, used for good chains, contains 14 parts of gold to 10 parts of alloy.

(2) Also used as a unit for measuring precious stones.
1 carat = 4 grains = about $3\frac{1}{8}$ Troy grains.
Since 1905, 1 carat = 200 milligrams, among international jewelers.

INDEX

(The numbers refer to pages.)

ANSWERS TO HART'S JUNIOR HIGH SCHOOL MATHEMATICS—BOOK I

Exercise 1. Pages 1–3

13. *a.* 44
b. 40
c. 40
d. 42
e. 38
f. 45
g. 40
h. 44
i. 50
j. 34
k. 42
l. 41

14. *a.* 274
b. 314
c. 337
d. 252
e. 362
f. 305
g. 354
h. 219
i. 282
j. 272

15. *a.* 2217
b. 3793
c. 2603
d. 1583
e. 2437.44
f. 206.83
g. 963.32
h. 1015.49

16. *a.* 4958
b. 4463
c. 20,404
d. 2652
e. 1887.50
f. 1932.48
g. 282.199
h. 2977.55

Exercise 2. Pages 4–5

1. $4,431.30
2. 264.75 lb.
3. $725.09
4. $7262.93
5. 2003 lb.
6. $412.50
7. $12,944.06

Exercise 3. Pages 5–6

1. *a.* 4237
b. 4156
c. 5023
d. 5637
e. 1714.44
f. 1691.751
g. 1535.633

2. 73
88
59
77
40
31
24
52

3. *a.* 5428
b. 15,323
c. 5779
d. 266.81
e. 70.908
f. 410.41
g. 306.118

4. 72
108
260
136
196
304
368
212

5. *a.* 52,726
b. 42,864
c. 347,760
d. 601,512
e. 435,150
f. 77,250
g. 578,550
h. 465,516

6. *a.* 42,176
b. 21,996
c. 10,450
d. 33,764
e. 69,525
f. 21,364
g. 49,932
h. 66,033

Exercise 4. Pages 7–9

1. Profit $25.27
2. Balance $2.72

3. Profit $297.85
4. Balance $26.70

5. Balance $52.69

Exercise 5

1. *a.* 7446
b. 6047
c. 6658
d. 5885
e. 5936
f. 5853
g. 5630
h. 7053
i. 6145

2. 89
78
46
37
65
60
31
14

3. *a.* 11,269
b. 5,717
c. 12,224
d. 80.595
e. 19.202
f. 25.812

4. 588
441
679
315
392
504
266
203

5.	Quot.	Rem.
	4	2;
	5	4;
	7	3;
	12	1;
	9	4;
	16	1;
	10	2;
	14	5;

6.	Quot.	Rem.
	2	8;
	3	7;
	5	0;
	8	1;
	6	4;
	10	7;
	6	8;
	9	8;

7.	Quot.	Rem.
	12	1
	10	6
	6	6
	5	5
	9	1
	8	4
	4	7
	2	6

8.	Quot.	Rem.
	12	0
	9	0
	13	6
	6	3
	8	0
	10	2
	5	3
	4	1

9. *a.* 116,519
b. 55,455
c. 82,368
d. 490.763
e. 75.394
f. 102.348

Exercise 6. Page 11

	1¢	5¢	10¢	25¢	50¢	1.00	2.00	5.00	10.00
1.	3	1	1		1				
2.	3		1			1			
3.		2		1			1	1	
4.	3		1			1			
5.	1			1	1	1		1	
6.	4	1		1		1	1		
7.	4		2	1					
8.	4	1	1	1	1	1			
9.	2	1	1	1		1			
10.		1				1		1	

Exercise 7. Pages 12–14

1. $ 7.10
2. $93.99
3. $6.39
4. .86
5. $180.03
6. $ 16.00

Exercise 9. Page 16

1. $2\frac{2}{5}$	4. $5\frac{5}{8}$	7. $5\frac{6}{3}$	10. $19\frac{3}{16}$	13. $25\frac{9}{3}$
2. $2\frac{3}{3}$	5. $4\frac{5}{4}$	8. $15\frac{1}{6}$	11. $26\frac{9}{12}$	14. $27\frac{1}{4}$
3. $4\frac{8}{5}$	6. $12\frac{3}{8}$	9. $22\frac{7}{7}$	12. $7\frac{5}{2}$	15. $6\frac{5}{3}$

16. $7\frac{1}{4}$	19. $12\frac{5}{6}$	22. $16\frac{3}{5}$	25. $15\frac{5}{12}$	28. $13\frac{15}{64}$
17. $5\frac{1}{7}$	20. $4\frac{3}{4}$	23. $23\frac{3}{4}$	26. $13\frac{9}{16}$	29. $10\frac{23}{25}$
18. $8\frac{3}{5}$	21. $6\frac{3}{10}$	24. $15\frac{7}{8}$	27. $11\frac{7}{32}$	30. $9\frac{19}{50}$

Exercise 10. Page 17

1. *a.* $\frac{4}{6}$ *b.* $\frac{6}{9}$ *c.* $\frac{10}{15}$ *d.* $\frac{16}{24}$ *e.* $\frac{20}{30}$
2. By 3.
3. *a.* $\frac{10}{15}$ *b.* $\frac{4}{6}$ *c.* $\frac{2}{3}$
4. By 7; $\frac{1}{10}$. Also by 14; $\frac{2}{5}$. Also by 2; $\frac{14}{35}$.
5. By 3; $\frac{15}{18}$. Equal to $\frac{5}{6}$; Law I.

Exercise 11. Page 18

1. $\frac{2}{3}$	6. $\frac{19}{33}$	11. $\frac{5}{8}$	16. $\frac{2}{5}$	21. $\frac{3}{7}$
2. $\frac{1}{3}$	7. $\frac{15}{32}$	12. $\frac{2}{7}$	17. $\frac{4}{6}$	22. $\frac{5}{15}$
3. $\frac{6}{17}$	8. $\frac{9}{13}$	13. $\frac{7}{10}$	18. $\frac{5}{13}$	23. $\frac{3}{5}$
4. $\frac{7}{16}$	9. $\frac{1}{3}$	14. $\frac{2}{3}$	19. $\frac{2}{5}$	24. $\frac{7}{8}$
5. $\frac{5}{6}$	10. $\frac{3}{8}$	15. $\frac{14}{31}$	20. $\frac{9}{17}$	25. $\frac{1}{1}$

Exercise 12. Pages 19–20

7. $2\frac{1}{40}$
8. $35\frac{5}{44}$
9. $\frac{3}{4}$
10. $\frac{1}{6}$
11. $\frac{3}{10}$
12. $\frac{7}{8}$
13. $1\frac{1}{24}$
14. 6
15. $\frac{5}{2}$
16. 9
17. 12
18. $\frac{1}{4}$
19. $15\frac{1}{4}$
20. $\frac{3}{8}$
21. 82
22. 5
23. 150
24. 150
25. 300
26. 2
27. 3
28. $2\frac{1}{6}$
29. 793
30. 396
31. 800.
32. 800
33. $8\frac{1}{3}$
34. $11\frac{3}{8}$
35. $562\frac{1}{2}$
36. $4640\frac{5}{8}$
37. $55\frac{11}{25}$
38. 66,500
39. 12,450

Exercise 13. Pages 20–21

1. 53¢
2. 26¢
3. 23¢
4. 48¢
5. $1.05
6. $1.58
7. 28¢
8. 45¢
9. 57¢
10. 79¢
11. $2.50
12. $1.84
13. $18.38
14. $9.56
15. 93¢
16. $1.46
17. $20.63
18. $2.08
19. 41¢
20. $1.23
21. $117.00
22. $3791.67
23. $10.64
24. $166.88
25. $504.38
26. $3.00
27. $5.00
28. $7.00
29. $2.17
30. $2.44

Exercise 14. Page 22

1. *a.* $\frac{3}{8}$ c cornmeal; $\frac{3}{4}$ c sugar; 2 t baking powder; $\frac{1}{4}$ t salt; $\frac{1}{2}$ c milk; $\frac{1}{2}$ egg; $\frac{1}{2}$ t melted butter.
   b. $\frac{1}{4}$ c cornmeal; $\frac{1}{2}$ c sugar; $1\frac{1}{3}$ t baking powder; $\frac{1}{6}$ t salt; $\frac{1}{3}$ c milk; $\frac{1}{3}$ egg; $\frac{1}{3}$ t melted butter.
2. *a.* $\frac{3}{4}$ c milk; $\frac{1}{4}$ c ground coffee; $\frac{1}{6}$ c sugar; $\frac{3}{8}$ t arrowroot; salt.
   b. 1 c milk; $\frac{1}{3}$ c ground coffee; $\frac{3}{8}$ c sugar; $\frac{1}{2}$ t arrowroot; salt.
3. *a.* $\frac{1}{3}$ t gelatine; $\frac{1}{12}$ c cold water; $\frac{1}{4}$ c boiling water; $\frac{1}{6}$ c sugar; $\frac{1}{4}$ c orange juice; $\frac{2}{3}$ t lemon juice; $\frac{1}{3}$ white of egg.
   b. $1\frac{1}{2}$ t gelatine; $\frac{3}{8}$ c cold water; $1\frac{1}{8}$ c boiling water; $\frac{3}{4}$ c sugar; $1\frac{1}{8}$ c orange juice; 3 t lemon juice; $1\frac{1}{2}$ white of egg.

4. *a.* 9 lb. sugar; 9 c water.
b. $6\frac{3}{4}$ lb. sugar; $6\frac{3}{4}$ c water.
c. $10\frac{1}{2}$ lb. sugar; $10\frac{1}{2}$ c water.

5. *a.* $\frac{7}{8}$ c flour;
$1\frac{1}{2}$ t baking powder;
1 egg;
$\frac{1}{2}$ t melted butter;
$\frac{1}{4}$ t salt;
$\frac{1}{2}$ c milk.

b. $4\frac{1}{12}$ c flour;
7 t baking powder;
$4\frac{2}{3}$ eggs;
$2\frac{1}{3}$ t melted butter;
$1\frac{1}{6}$ t salt;
$2\frac{1}{3}$ c milk.

Exercise 15. Pages 24–25

1. $3\frac{1}{3}$
2. $2\frac{2}{3}$
3. $3\frac{3}{4}$
4. $2\frac{1}{3}$
5. $\frac{2}{3}$
6. $1\frac{1}{4}$
7. 6
8. $11\frac{1}{5}$
9. 10
10. $\frac{5}{6}$
11. $\frac{3}{32}$
12. $\frac{5}{18}$
13. $21\frac{1}{3}$
14. $1\frac{5}{8}$
15. $\frac{3}{16}$
16. 64
17. $1\frac{4}{5}$
18. $\frac{5}{12}$
19. $2\frac{1}{8}$
20. $1\frac{1}{8}$
21. $2\frac{2}{7}$
22. $\frac{2}{5}$
23. $\frac{7}{27}$
24. $4\frac{1}{6}$
25. $4\frac{1}{5}$
26. 8
27. $3\frac{1}{5}$
28. $7\frac{1}{5}$
29. $4\frac{1}{2}$
30. $5\frac{1}{4}$
31. 6
32. 20
33. 8
34. $6\frac{2}{3}$
35. *a.* 25
b. Divide total quantity by quantity required for one.
36. 12
37. 6
38. 16
39. 4
40. 6 pcs.
41. 30 pcs.
42. 54 pcs.
43. 6 pcs.
44. 20 wk.
45. $94\frac{1}{2}$ da.
46. 18 da.
47. 3 gal.
48. 20 tons.

Exercise 16. Page 26

1. *a.* $\frac{8}{12}$ *b.* $\frac{12}{18}$ *c.* $\frac{6}{9}$ *d.* $\frac{16}{24}$
2. *a.* $\frac{15}{25}$ *b.* $\frac{21}{35}$ *c.* $\frac{60}{100}$ *d.* $\frac{33}{55}$
3. *a.* $\frac{10}{12}$ *b.* $\frac{30}{36}$ *c.* $\frac{20}{24}$ *d.* $\frac{35}{42}$ *e.* $\frac{80}{96}$
4. *a.* $\frac{32}{8}$ *b.* $\frac{24}{6}$ *c.* $\frac{40}{10}$
5. $\frac{147}{21}$; $\frac{98}{14}$; $\frac{196}{28}$
6. $\frac{72}{9}$; $\frac{32}{4}$; $\frac{64}{8}$; $\frac{128}{16}$
7. $\frac{15}{3}$; $\frac{50}{10}$; $\frac{35}{7}$

8. *a.* $3\frac{6}{9}$ *b.* $6\frac{6}{9}$ *c.* $15\frac{6}{9}$
9. *a.* $6\frac{6}{12}$ *b.* $9\frac{6}{12}$ *c.* $8\frac{6}{12}$ *d.* $60\frac{6}{12}$
10. *a.* $19\frac{6}{16}$ *b.* $13\frac{6}{16}$ *c.* $8\frac{6}{16}$ *d.* $32\frac{6}{16}$
11. $13\frac{6}{32}$ 13. $19\frac{6}{25}$
12. $13\frac{6}{18}$ 14. $25\frac{6}{30}$

Exercise 17. Pages 27–28

1. $\frac{5}{6}$
2. $\frac{3}{4}$
3. $\frac{7}{12}$
4. $1\frac{1}{6}$
5. $1\frac{1}{12}$
6. $1\frac{1}{12}$
7. $1\frac{3}{15}$
8. $1\frac{1}{10}$
9. $1\frac{3}{8}$
10. $1\frac{7}{15}$
11. $1\frac{17}{24}$
12. $1\frac{5}{16}$
13. $1\frac{11}{24}$
14. $1\frac{1}{24}$
15. $1\frac{11}{20}$
16. $1\frac{1}{2}$
17. $\frac{1}{3}$
18. $1\frac{9}{10}$
19. $\frac{1}{12}$
20. $\frac{3}{10}$
21. $1\frac{3}{24}$
22. $4\frac{5}{8}$
23. $1\frac{3}{4}$
24. $4\frac{9}{10}$
25. $5\frac{7}{8}$
26. $7\frac{5}{9}$
27. $\frac{1}{24}$
28. $\frac{1}{32}$
29. $5\frac{3}{4}$
30. $7\frac{11}{12}$
31. $13\frac{7}{8}$
32. $12\frac{1}{10}$
33. $6\frac{13}{14}$
34. $11\frac{39}{40}$
35. $17\frac{3}{8}$
36. $12\frac{19}{21}$
37. $3\frac{5}{10}$
38. $4\frac{1}{6}$
39. $5\frac{5}{12}$
40. $12\frac{17}{24}$
41. $17\frac{7}{12}$
42. 29
43. $5\frac{14}{15}$
44. $14\frac{1}{12}$
45. $1.07
46. $51.56
47. $11\frac{3}{8}$ ft.
48. 23 in.
49. 23 in.
50. $2\frac{1}{8}$ loss

Exercise 18. Pages 28–29

1. $1.47
2. $1.70
3. $9.02
4. $2.59
5. $2.01
6. $2.34
7. $4.28
8. *a.* $6.70 *b.* $0.62 *c.* $6.08
9. $6.56
10. 17.49\frac{1}{2}$

Exercise 19. Page 30

1. $10.10
2. $17.67
3. *$4.03*
4. *$4.60*
5. *1.12\frac{2}{3}$* or $1.13
6. $7.83
7. $23.71
8. $15.89
9. $8.05
10. $2.36 more

Exercise 20. Pages 31–32

9. *c.* It does not change the value.
10. *d.* One zero: divides it by 10.
Two zeros: divides it by 100.
Three zeros: divides it by 1000.

Exercise 21. Pages 33–35

1. *a.* 3.99 *b.* .340 *c.* 9.36 *d.* 26.08
e. 41.445 *f.* 14.88 *g.* 76.437
2. *a.* .8303 *b.* 1.0512 *c.* 16.53 *d.* .15246
e. .15822 *f.* 9.62678 *g.* 24.52175
3. *a.* 25.7054 *b.* .07320 *c.* 1.53714 *d.* 23.544
e. 30.20682 *f.* .46546 *g.* 41.48424
4. *a.* 78; 20.45; 9254.3; 672.57; 82.96
b. One place to the right.
5. *a.* 352; 186; 4523.7; 6806.4; 7480.95
b. Two places to the right.
6. *a.* 3257; 53279; 47081; 25860; 94300
b. Three places to the right.
7. 1320 ft.
8. 468.875 sq. yd.
9. $284.17
10. $505.08
11. $3.15
12. 225.75 in.
13. $1599
14. $14,282.97
15. $4180.13
16. $13,605.90
17. $2086.58
18. $3388.65
19. $1063.13
20. Multiply the cost of one article by the number of the articles.
21. *a.* 1203.75 lb.
b. 57.5 lb.
c. about .08 lb.
22. 203.928 ft.
23. *a.* 32 ft. 8+ in.
b. 3280⅝ ft.; 112⅝ ft.
24. *a.* 94.248 in.
b. 785.4 ft.

Exercise 22. Page 36

1. .12
2. .93
3. *9.15*
4. *3.142*
5. .023
6. .085
7. 1.184
8. 1.139
9. 1.454
10. 6.903
11. .274
12. 4.023

Exercise 23. Pages 38–39

1. *a.* 69 *b.* 8.1 *c.* .74 *d.* 10.9 *e.* 34.2
2. *a.* 24 *b.* 897 *c.* 2.9 *d.* .138 *e.* 165.2
3. 3.1
4. 52.8
5. 3.07
6. 64
7. 695.3
8. 6.294
9. 2650
10. 3.25
11. 23.51
12. 3.6
13. 1.21
14. 78
15. 4.63
16. 8.2
17. 245.63
18. 40.26
19. 10.51
20. 300.5
21. *a.* 9.25 .867 5.43 65.42 .072 .0065
b. Move the decimal point one place to the left.
22. *a.* .4357 .3625 .847 2.951 .0643 .0086
b. Move the decimal point two places to the left.
c. Move the decimal point three places to the left.
23. .137
24. 23.15
25. 2.6321
26. 2.8921
27. 1.743
28. .5171
29. 11.5571
30. 18.43
31. 1.13
32. 1.82
33. 8.584
34. 1.1789
35. 3.8575

Exercise 24. Page 39

1. 208.84
2. 309.26
3. 7.48
4. 2.34
5. 5.08
6. 9.64
7. 284.53
8. 70.48
9. 37.93
10. 51.91
11. 188.63
12. 44.16
13. 393.90
14. 2.29
15. 11.63

Exercise 25. Pages 40–42

1. 27.5 bu.
2. 164.3 lb.
3. 2.5 t
4. $2.53
5. $3.98
6. $268.89
7. $208.58
8. 351.1 lb.
9. 79.5
10. 2.5 lb.; .13 lb.
11. 13.2 mi.
12. 28.9¢ per day
13. *1.3¢*
14. *270.6 da.*
15. 32.9 mi.
16. 37.5 bu.
17. 33.2 bu.
18. $533.33
19. .797

Exercise 26. Pages 42–43

1. 1361 mi.
2. 272.2 mi.
3. 91.2 mi.
4. 715.2 ft.
5. 18,912 mi.
6. 1.8 times
7. 4.4 times
8. 4.8 times
9. 3.0 times

Exercise 27. Pages 43–44

1. .6
2. .625
3. .28
4. .275
5. .65
6. .54
7. .833
8. .875
9. .666
10. 3.75
11. .272
12. .416
13. .187
14. .156
15. 7.666
16. 11.8
17. 5.444
18. 6.75
19. 27.166
20. 19.916
21. 28.25
22. 16.70
23. 21.980
24. 8.975
25. 29.90
26. 15.1
27. 63.675
28. 12.25
29. 137.375
30. 31.500
31. 359.375
32. 335.6500

Exercise 28. Pages 44–45

1. 1.7¢
2. .3¢
3. 1.8¢
4. 3.8¢
5. 42.94¢
6. $5.89
7. $18.90
8. 59¢
9. $19.49
10. *a.* 29.4 mi. *b.* 20.6 mi.

Exercise 29. Pages 45–46

1. 26.274
2. 157.95
3. 2.07
4. 20
5. $13.14
6. 51.625
7. 2.625 yd.
8. $1.89
9. 142.8 lb.
10. *a.* $13.80 *b.* 69¢

Exercise 30. Pages 46–47

1. ⅛; ¼; 1/16; ⅓; ⅝; 7/20
2. $30,330.67
3. 1/144 mi.
4. *102.5 d*
5. *$96.28*
6. 4279.1 lb.
7. *a.* 386.10; *b.* $7\frac{1}{27}$
8. $1565\frac{3}{60}$

Exercise 31. Page 47

1. $42.39
2. $20.04
3. $6.63
4. $55.04
5. $124.10

Exercise 32. Page 48

1. .07	10. .25	19. .15	28. 65%
2. .09	11. .85	20. .35	29. 49%
3. .05	12. .72	21. .60	30. 24%
4. .02	13. .91	22. .16	31. 72%
5. .11	14. .63	23. .66	32. 55%
6. .24	15. .13	24. .80	33. 98%
7. .19	16. .10	25. .75	34. 43%
8. .01	17. .40	26. 17%	35. 86%
9. .33	18. .70	27. 28%	

Exercise 33. Pages 49–52

1. 11; 15.00; 24; 166; 135
2. 38.50; 52.50; 84; 581; 472.50
3. 52.00; 256; 424; 1456.
4. 65; 320; 530; 1820
5. 52.35; 840; 42.75; 145.5
6. $1.62; $2.88; $7.50; $15.00
7. $16.25; $20.80; $1.90; $3.15
8. 9.24; 8.25; 10.78; 14.63

9. 143.64	16. 3.4605	23. 14 girls; 21 boys
10. 103.14	17. 3.432	24. The first school
11. 545.44	18. 83	25. 104; 696
12. $1950.50	19. 570	26. 250
13. $44.08	20. 9100	27. $417.50
14. $57.51	21. —	28. $206.25
15. $2.51	22. 450 pupils	29. *a.* $5; *b.* $100; *c.* $30
30. 6 pupils	34. *a.* $113.25	*b.* $213.75
31. $1935.00	*c.* $277.50	*d.* $360.00
32. $7983.80	35. 14 lb.	
33. 171 lb.	36. $337.00	

37. 1300 lb. copper; 700 lb. zinc.

38. 42,306,200 people in cities and towns
49,663,800 people in the country.
39. $215.33
40. 400,000 in national guard
520,000 in regular army
3,080,000 in national army

Exercise 34. Page 52

1. $\frac{1}{2}$	6. $\frac{4}{5}$	11. $\frac{1}{20}$	16. $\frac{1}{50}$	21. $\frac{3}{25}$
2. $\frac{1}{5}$	7. $\frac{3}{10}$	12. $\frac{7}{20}$	17. $\frac{11}{50}$	22. $\frac{7}{25}$
3. $\frac{3}{5}$	8. $\frac{7}{10}$	13. $\frac{17}{20}$	18. $\frac{1}{25}$	23. $\frac{13}{20}$
4. $\frac{9}{10}$	9. $\frac{1}{4}$	14. $\frac{3}{20}$	19. $\frac{4}{25}$	24. $\frac{1}{100}$
5. $\frac{3}{5}$	10. $\frac{3}{4}$	15. $\frac{9}{20}$	20. $\frac{8}{25}$	25. $\frac{19}{25}$

Exercise 35. Page 53

1. 50%	11. 75%	21. 50%	31. $\frac{1}{30}$
2. $33\frac{1}{3}$%	12. 40%	22. 25%	32. $\frac{1}{8}$
3. 25%	13. $83\frac{1}{3}$%	23. 50%	33. $16\frac{2}{3}$%
4. 20%	14. $37\frac{1}{2}$%	24. 20%	34. $\frac{7}{8}$
5. $16\frac{2}{3}$%	15. $62\frac{1}{2}$%	25. 25%	35. $\frac{3}{5}$
6. $12\frac{1}{2}$%	16. 80%	26. $\frac{1}{2}$	36. $\frac{3}{8}$
7. 10%	17. $87\frac{1}{2}$%	27. $\frac{1}{6}$	37. $83\frac{1}{3}$
8. $8\frac{1}{3}$%	18. 60%	28. $\frac{1}{4}$	38. $\frac{3}{4}$
9. $6\frac{1}{4}$%	19. 75%	29. $12\frac{1}{2}$%	39. $\frac{2}{3}$
10. $66\frac{2}{3}$%	20. $66\frac{2}{3}$%	30. 20%	40. $\frac{1}{2}$

Exercise 36. Pages 54–56

1. 90; 1600; 375; 3200; 130
2. 200; 500; $171; $6.15
3. $95; $2.36; $2.50; 2.4; 1.6
4. $3; $9; $150; $3.30
5. 30; 150; 1200; 225 lb.; 18 t.
6. 10 mi.; 26 bu.; $14.20; 72 d.
7. $10; $50; $90; 3; 4.5
8. 60 mi.; 150 ft. 250; $5
9. $1200; 5060; $246.20; 15.6
10. $240; 2100; 2730; 6.15
11. 50 da.; 1120; $44.22
12. 7210; $39.90; 21,000 sq. ft.
13. *$225.15; 9 mi.;* 376.473
14. *90*

15. 1245 bu.
16. 6 d. clear; 24 d. cloudy.
17. 720
18. *a.* \$.50; *b.* \$.70⅚ or \$.71; *c.* \$90
19. Colorado Springs, ⅜ Everett, Mass., ⅜
Decatur, Ill., ½ Lexington, Ky., ⅓
Madison, Wis., 1/30 Portland, Me., 1/6
Roanoke, Va., ⅝ Shreveport, La., ¾
Somerville, Mass., ¼ Yonkers, N. Y., ⅜
20. 475,822,500. **21.** 1,618,066
22. Colonel, \$4000
Lieut. Col., \$3500
Major, \$3000
Captain, \$2400
First Lieut., \$2000
Second Lieut., \$1700
23. Cincinnati team, 5 games
Chicago team, 3 games
24. *a.* Won, 6 games; lost, 25%
b. 5 games
c. Won, 3 games; lost, 2 games; tied, 1 game.
25. 1¢

Exercise 37. Page 57

1. Twice
2. Five times
3. Three times
4. Seven times
5. Ten times
6. Eight times
7. One and one-half times
8. Three and one-fourth times
9. Four and three-fourths times
10. Five and one-tenth times
11. Eight and one-fifth times
12. Nine and one-third times

13. 3.15	**15.** 6.42	**17.** 1.13	**19.** 1.45	**21.** 8.32
14. 2.73	**16.** 5.18	**18.** 6.27	**20.** 2.16	**22.** 5.49

23. 500% 1000%
24. 700% 900%
25. 600% 1100%
26. 450%
27. 225%
28. *312½%*
29. *133⅓%*
30. *575%*
31. 666⅔%
32. 137½%
33. 262½%
34. 316⅔%
35. 483⅓%

Exercise 38. Pages 58–59

2. .19; 1.23
4. Twice to twenty times their money.
5. 1575 pupils
6. $2800
7. $11.25
8. $13.50
9. 63.867 or 64¢
10. 15.010¢ or 15¢
11. 8¢
12. 40.5¢ or 41¢
13. $1.195 or $1.20
14. 38.88¢ or 39¢
15. 28.944¢ or 29¢

Exercise 39. Pages 59–61

1. .005; .025; .035; .055.
2. .0025; .0125; .0325; .0525.
3. .002; .022; .032; .042.
4. .00125; .01125; .03125; .05125.
5. .0075; .0175; .0375; .0475.
6. .004; .014; .044; .034.
7. .00375; .02375; .04375; .05375
8. .001; .021; .031; .041.
9. .003; .013; .053; .043.
10. .007; .027; .037; .057.

11. $227.50
12. $123.53
13. $82.13
14. $48.88
15. $226.10
16. $36.56
17. $220.00
18. $29.11
19. $26.69
20. $19.38
21. $275.00
22. $21.70
23. $78.75
24. $71.25
25. $23.38
26. $3.13
27. 10,400 lb. lead; 320 lb. silver.

28 8.1 in.

29. Cow No. 1, 351.39 lb.; $168.67.
Cow No. 2, 261.196 lb.; $125.37.
Cow No. 3, 251.2161 lb.; $120.58.

30. 6,530,012, in New England States.
31,546,396, in Atlantic States.
29,890,900, in North Central States.
17,198,764, in South Central States.
2,575,216, in Mountain States.
4,230,712, in Pacific States.

Exercise 40. Pages 62–63

1. $628.80
2. $27.45
3. $131.25
4. 417.6
5. 460
6. 6039
7. $1512.50
8. $23.74
9. $60.50
10. $224.19
11. $38.28
12. 13.58
13. 208.6
14. 28.5
15. $1590

Exercise 41. Pages 63–64

1. *a.* 3932
   b. 5665
   c. 4499
   d. 4691
   e. 4245
   f. 4456
   g. 3976
2. $230\frac{5}{24}$
3. $5\frac{1}{5}$
4. $17\frac{5}{8}$
5. $24\frac{11}{30}$
6. *a.* 2 pk.
   b. 1000 lb.
   c. 2 ct.
   d. 80 sq. rd.
   e. 160 rd.
   f. 12 hr.
7. *a.* 1 pt.
   b. 250 lb.
   c. 1 pt.
   d. 20 sq. rd.
   e. 40 rd.
   f. 3 hr.
8. 6.6 t.
9. $1,062,500
10. $390,542.60
11. *a.* 50%; $33\frac{1}{3}$%; 25%; 20%; $16\frac{2}{3}$%; $12\frac{1}{2}$%; 10%; $8\frac{1}{3}$%.
    b. 350%; 225%; 550%; 620%; 275%.
12. 3 times; 5 times; $2\frac{1}{2}$ times; $1\frac{3}{4}$ times.

Exercise 42. Pages 64–65

1. *a.* 7873 mi.
   b. 5800 mi.
   c. 5800 mi.
2. 1915, 1051;
   1916, 732;
   1917, 1682.
3. *a.* 3196.9 t.
   b. 4613.9 t.
4. *a.* 3211.4 t
   b. 3150.3 t
5. *a.* $3853.68
   b. $3780.36
6. *a.* $2311.21
   b. $2268.22

Exercise 43. Pages 67–69

1. 682.50
2. 487.60
3. 540
4. 675
5. *632*
6. *1260*
7. *240*
14. 90.30
15. *a.* 66¢ *b.* 81¢ *c.* $1.17 *d.* $3.25 *e.* 61¢
16. *a.* $4.36 *b.* $7.80 *c.* $2.52 *d.* $1.48 *e.* $1.51
17. 2210
18. $7959.69
19. $165.00
20. $10,062.50

8. 42
9. 15
10. 35.52
11. 77.35
12. 83.20
13. 40.5
21. 1004
22. $958.75
23. *a.* $178.20 *b.* $135.00 *c.* $81.00
24. $11.68
25. *a.* 56,375 lb. *b.* 79.95 lb.

Exercise 44. Pages 70–71

1. *a.* $2.00 *b.* $2.25 *c.* $2.50 *d.* $2.30
2. *a.* 20¢ *b.* 35¢ *c.* 75¢ *d.* 60¢
3. Gross profit = $1.00; net profit = 75¢.
4. *a.* $2.50 *b.* $2.25 *c.* $1.10 *d.* $0.80 *e.* $0.49
5. *a.* 63¢ *b.* $1.05 *c.* 84¢ *d.* 25¢ *e.* 48¢
6. *a.* 52¢ *b.* $1.60 *c.* 96¢ *d.* $2.20 *e.* $3.80
7. *a.* $2.63 *b.* $6.30 *c.* $12.94 *d.* $8.61 *e.* $10.67
8. $9.77
9. $1437.50
10. $300
11. $7.30
12. $12.51
13. $15.50
14. $1.29
15. $1.14
16. $1.44
17. $2.55
18. $3.21
19. $406.88
20. ———
21. $5625
22. $627.42
23. $6800
24. *a.* $2.25 *b.* $2.70 *c.* $.72 *d.* $45.00 *e.* $.54
25. *a.* $3.75 *b.* $2.70 *c.* $2.06 *d.* $.51 *e.* $.36

Exercise 45. Pages 72–74

4. *a.* $0.18; $1.62 *b.* $0.25; $2.25 *c.* $0.34½; $3.11
   d. $0.08; $.72 *e.* $0.05; $.45
5. *a.* 28¢; 56¢ *b.* 45¢; 90¢ *c.* $0.90; $1.80
   d. $0.50; $1.00 *e.* $2.50; $5.00
6. *a.* $3.00; $9.00 *b.* $5.75; $17.25 *c.* $3.93¾; $11.81
   d. $3.20; $9.60 *e.* $2.41; $7.24
7. Suit, $54.17; overcoat, $58.33.
8. $2.22
9. *a.* $20.03; *b.* $22.37; *c.* 41.04¢ or 41¢.
10. $6.06
11. $1.90
12. $51 to dealer; $60 to ordinary purchaser.
13. $36.40
14. $1706.25
15. *a.* $52.50; *b.* $44.62; *c.* $7.13

Exercise 46. Pages 75–77

1. *a.* $15.00; *b.* $60.00
2. *a.* $656.25; *b.* $164.07
3. *a.* $32.50; *b.* $277.50
4. *a.* $775; *b.* $800.35; *c.* $14,699.65
5. *a.* $278.36; *b.* $1577.39
6. *a.* $39.35; *b.* 1895.65
7. $2810.83
8. $162.38
9. $1187.50
10. *a.* $696.00; *b.* $1596.00; *c.* $133.00
11. $63

Exercise 47. Page 77

1. $58\frac{13}{30}$
2. $73\frac{1}{3}$
3. $155\frac{3}{8}$
4. $10\frac{3}{7}$
5. $22\frac{11}{20}$
6. 82.94; 332.85
7. *a.* 29.31
   b. 9.22
8. 50%
   $62\frac{1}{2}$%
   $33\frac{1}{3}$%
   20%
   $16\frac{2}{3}$%
   90%
9. 3.25
   6.14
   .025
   .045
10. $8.925 or $8.93
    $12.75
    $33.75

Exercise 48. Pages 77–79

1. $2.70;
   profit, 30¢ per box;
   total profits, $37.50.
2. 52,486
3. 39,000 not obtained;
   65,000 poorly trained.
4. $7.20
5. Food, $774;
   rent, $324;
   clothing, $234;
   fuel and light, $99;
   misc., $369
6. $8.83
7. $2,240,000,000
8. *a.* $18.99;
   b. $59.40
9. Cost, $23.76;
   profit, $5.94
10. $245
14. $13.39
15. $13.13

Exercise 49. Pages 82–84

1. 33⅓%
2. 12½%
3. 50%
4. 20%
5. 25%
6. 10%
7. 25%
8. 16⅔%
9. 33⅓%
10. 12½%
11. 50%
12. 25%
13. 62½%
14. 25%
15. 66⅔%
16. 33⅓%
17. 20%
18. 20%
19. 33⅓%
20. 66⅔%
21. 37½%
22. 16⅔%
23. 50%
24. 14¾%
25. 300%
26. 200%
27. 550%
28. 233⅓%
29. 420%
30. 312½%
31. 5%; 7%
32. 3%; 7%
33. 5%; 1%
34. 12½%; 33⅓%
35. 2½%
36. 25%
37. 5%
38. 3⅛%
39. 25%
40. 12½%
41. 16⅔%; 20%
42. 40%; 37½%
43. *a.* Boys, 60%; girls, 40%
    b. 55%
    c. Greater.
44. Lowest class, 12%
    other classes, 88%
45. 40%
46. 66⅔%
47. *a.* 75%
    b. The second boy.
48. 20%
49. 4%
50. 3⅛%

Exercise 50. Page 84

1. 3%
2. $40
3. 25%
4. 174.6
5. 112½%
6. 71.15
7. 150%
8. 133⅓%
9. $906.56
10. 95%

Exercise 51. Pages 85–87

1. 60%
2. 25%
3. *a.* 20%
   b. 60%
4. *a.* 20%
   b. 100%
   c. 66⅔%
5. *a.* 66⅔%
   b. 166⅔%
6. *a.* 33⅓%
   b. 66⅔%
7. 13⅓%
8. 100%
9. 104+%
10. 109.9%
11. 108.3%
12. 180.9%
13. *a.* 20%
    b. 60%
14. 25%
15. 3.7%

Exercise 52. Pages 88–90

1. 33⅓%
2. *a.* $1.20
   b. 23⅓%
3. 16⅔%
4. *a.* $1.80
   b. 8⅓%
5. *a.* 3¾%
   b. 9.3%
6. 6⅔%
7. 12½%
8. 10%
9. 66⅔%
10. 8.1%
11. *a.* $1.00
    b. 12%
12. 10.9%
13. *a.* $1.40
    b. 62.1%
14. *a.* $3.23
    b. 24.1%
15. *a.* $8.50
    b. $15.00
    c. 43⅓%

Exercise 53. Pages 91–93

4. *a.* $1.00
   b. 33⅓%
   c. 25%
5. *a.* $2.00
   b. 40%
   c. 66⅔%
6. Profit, 10%
7. Profit, 33⅓%
8. Profit, 20%
9. Loss, 16⅔%
10. Loss, 10%
11. Profit, 20%
12. Profit, 100%
13. Profit, 200%
14. *a.* 14¢
    b. 20%
    c. 7¢
    d. 10%
15. *a.* Profit, 12½%
    b. 4¢
    c. 6¼%
16. 12½%
17. 33⅓% of cost;
    25% of selling price.
18. 40%
19. 36%
20. 6½% on one;
    5½% on other.
21. 74%, gross profit.
    45%, if some are spoiled.
22. 60%

Exercise 54. Pages 94–95

1. 1500
2. 700
3. 400
4. 7000
5. 600
6. 810
7. $203
8. 4060
9. 5500
10. 6400
11. $450
12. $10
13. $39.75
14. 67.2
15. 52
16. $187.20
17. $50
18. 50
19. 1575 ft.
20. $325
21. $8340

Exercise 55. Page 96

1. 110%
2. 118%
3. 94%
4. 75%
5. 120%
6. 300
7. $700
8. 435
9. 92
10. $168
11. $250
12. 120
13. 200
14. $450
15. $300

Exercise 56. Pages 96–99

3. $126.15
4. $7104
5. 110%
6. .005; .035; 2.25; 3.50
7. $9.59
8. $959.00
9. 5%; 2%; 1%
10. *a.* Adding to it.
    b. Subtracting from it.
    c. 118%
    d. 92½%
11. 20%
12. $122.17
13. $10,080
14. 12½%
15. 11⅑%
16. $925
17. $372
18. 300%
19. 400%
20. 400%
21. *a.* $4.30
22. 5%
23. Won, 62½%; lost, 37½%.
24. 57⅐% less.
25. 12½%
26. First class, 50% second class, 16⅔%.
27. 28%
28. $4665
29. $2900
30. $2.50
31. Final price, $33.91; profit, $5.41.

Exercise 57. Pages 101–102

1. $30
2. $60
3. $15
4. $45
5. $75
6. $120
7. $830
8. Int., $170.63; amount, $1145.63.
9. Int., $9.38; amount, $634.38.
10. Int., $128; amount, $1328.
11. Int., $6.88; amount, $281.88.
12. Int., $20; amount, $1520.
13. Int., $18.75; amount, $1518.75.
14. Int., $16.90; amount, $392.40.
15. Int., $1.70; amount, $86.70.
16. $45
17. $2.50
18. 25¢
19. *$120*
20. *$325*

Exercise 58. Page 103

1. Int., $5.75; amount, $580.75.
2. Int., $37.33; amount, $837.33.
3. Int., $2.71; amount, $652.71.
4. Int., $6.25; amount, $381.25.
5. Int., $27.24; amount, $522.24.
6. Int., $23.33; amount, $273.33.
7. Int., $36.58; amount, $421.58.
8. Int., $.63; amount, $150.63.
9. Int., $15.58; amount, $440.58.
10. Int., $36.46; amount, $1286.46.
11. $33.75
12. $5.00
13. $1.35
14. $15.63
15. $7.50

Exercise 59. Pages 104–105

1. Int., $1.13; amount, $151.13.
2. Int., $.76; amount, $275.76.
3. Int., $1.17; amount, $351.17.
4. Int., $1.89; amount, $426.89.
5. Int., $5.25; amount, $1505.25.
6. Int., $.31; amount, $225.31.
7. Int., $.53; amount, $175.53.
8. Int., $1.21; amount, $331.21.
9. Int., $5.00; amount, $4005.00.
10. Int., $52.50; amount, $3052.50.
11. Int., $45.14; amount, $2545.14
12. Int., $1.88; amount, $126.88.
13. Int., $3.58; amount, $218.58.
14. Int., $102.93; amount, $3302.93.
15. Int., $304.17; amount, $7804.17.
16. Int., $43.75; amount, $3543.75.
17. Int., $8.25; amount, $458.25.
18. Int., $24.30; amount, $524.30
19. Int., $42.00; amount, $1542.00.
20. Int., $171.74; amount, $996.74.

Exercise 60. Page 106

1. 2 yr., 5 mo., 6 d.
2. 2 yr., 5 mo., 6 d.
3. 1 yr., 7 mo., 13 d.
4. 2 yr., 5 mo., 24 d.
5. 1 yr., 1 mo., 21 d.
6. 10 mo., 21 d.
7. 7 mo., 14 d.
8. 4 mo., 6 d.
9. 1 yr., 8 mo., 15 d.
10. 2 mo., 22 d.
11. 205 d.
12. 84 d.
13. 82 d.
14. 109 d.
15. 177 d.

Exercise 61. Pages 107–108

1. Int., \$7.64; amount, \$242.64.
2. Int., \$38.10; amount, \$688.10.
3. Int., \$23.63; amount, \$1523.63.
4. Int., \$86.66; amount, \$2586.66.
5. Int., \$91.00; amount, \$966.00.
6. Int., \$.51.
7. \$4.72.
8. \$.30
9. \$3515.51
10. \$1340.91

Exercise 62. Pages 110–111

3. \$437.75
4. .....
5. \$2100
6. *b.* \$964.25
   c. \$3315
7. *b.* \$2050
8. *b.* \$129.69
9. *b.* \$827.73
10. *b.* \$1137.03

Exercise 63. Pages 113–114

6. Yes.
7. *a.* Straight line.
   b. Yes.
9. *a.* No.
   c. Parallel lines.

Exercise 64. Pages 115–119

9. 50%; 200%
10. 25%; 400%
11. *b.* *33⅓%;*
    c. *300%*
13. *b.* MN, 2 in.;
    RS, 2½ in.
    XY, 1 in.
14. *a.* 125%
    b. 40%
    c. 50%
    d. 200

Exercise 65. Pages 120–122

1. Wheat, 60 lb.;
   oats, 32 lb.
   shelled corn, 56 lb.;
   corn on cob, 72 lb.
   barley, 48 lb.;
   rye, 56 lb.

2. *a.* $\frac{7}{9}$
   b. oats, $\frac{8}{15}$;
   rye, $\frac{14}{15}$;
   barley, $\frac{4}{5}$
   c. oats, $66\frac{2}{3}$%;
   wheat, 120%.
   shelled corn, $116\frac{2}{3}$%;
   corn on cob, 150%.

3 *b.* CD, $1\frac{1}{2}$ in.;
   c. EF, $2\frac{1}{4}$ in.;
   d. HG, $\frac{7}{8}$ in.;
   e. IJ, $\frac{5}{8}$ in.;
   f. KL, $1\frac{3}{16}$ in.;
   g. MN, $2\frac{5}{8}$ in.

4. *b.* CD, $1\frac{1}{4}$ in.;
   c. EF, $3\frac{3}{8}$ in.;
   d. GH, $\frac{3}{4}$ in.;
   e. IJ, $1\frac{7}{8}$ in.;
   f. KL, $3\frac{1}{2}$ in.;
   g. MN, $4\frac{3}{8}$ in.

Exercise 66. Pages 122–125

2. The sum.
3. 2; 0; 1.
6. *a.* $\angle 2$
   b. $\angle 1 = \angle 3$; $\angle 1 < \angle 4$
   c. $\angle 2 > \angle 3$; $\angle 2 < \angle 4$
   d. $\angle 3 < \angle 4$
8. *c.* Right angle
9. *b.* Smaller
10. *b.* Larger

Exercise 67. Pages 125–128

4. *b.* No; vertical.
5. *a.* Parallel lines.

Exercise 68. Pages 128–129

2. *a.* No.

Exercise 69. Pages 129–130

1. 45°
2. 30°
3. $22\frac{1}{2}$°
4. 18°
5. 27°
6. 30°
7. 15°
8. 10°
9. $45\frac{5}{9}$°
10. $38\frac{8}{9}$°
11. 90°
12. 60°; 30°
13. 120°
14. 150°
15. 360°
270°
216°

Exercise 72. Pages 136–138

1. *a.* Right angles
b. Equal
c. Equal
d. Perpendicular
e. Parallel
2. *f.* Rectangle
3. *d.* The diagonals of a rectangle are equal.
f. Each diagonal of a rectangle divides the other into two equal parts.
4. *c.* A diagonal of a rectangle does not divide the angles through which it is drawn into two equal parts.

Exercise 73. Pages 138–140

1. *a.* Right angles.
b. Opposite sides parallel. All sides equal.
2. *d.* The diagonals of a square are equal.
3. *b.* Each diagonal of a square divides the other into two equal parts.
4. *b.* The two diagonals of a square form right angles. The diagonals of a square are perpendicular lines.
5. *c.* A diagonal of a square does divide the angles through which it is drawn into two equal parts

6. *a.* No. Yes.
b. Yes. Yes.
c. No. Yes.
d. No. Yes.

7. 45°
8. Yes.
9. Not a square.
10. A square.

Exercise 74. Pages 140–145

1. *a.* 2 in.;
b. ½ in.;
c. 3 in.;
d. 1½ in.;
e. 60 ft.

2. *a.* 1½ in.;
b. ¼ in.;
c. 2 in.;
d. ⅞ in.;
e. 1⅝ in.

3. *a.* ¾ in.;
b. 1¼ in.;
c. ⅜ in.;
d. ⅛ in.;
e. $\frac{15}{16}$ in.;
f. 40 ft.

4. About 87¼ ft.
5. *b.* About 127 ft.
c. Same as *b.*
8. *c.* 8⅝ in., wide.
11⅝ in., long.
11. *c.* 15 in., wide.
17 in., long.

Exercise 75. Pages 146–147

1. 12 sq. in.
2. *a.* 30 sq. in.;
b. 48 sq. ft.;
c. 63 sq. yd.;
d. 33 sq. rd.;
e. 18 sq. mi.
3. ½
6. 10
7. 3¾

Exercise 76. Pages 147–150

1. 925 sq. ft.
2. 4320 sq. ft.
3. 33 sq. ft.
4. 200 sq. in.
5. 372 sq. yd.
6. 198.95 sq. rd.
7. 528.125 sq. yd.
8. 76.96 sq. ft.
9. 659⅜ sq. ft.
10. 48⅛ sq. in.
11. 198 tiles.
12. 108 sq. ft.
13. 72 sq. ft.
14. $280.50
15. $69.83
16. $2\frac{17}{32}$ a.
17. $52.44
18. 75¢
19. $224.25
20. $164.00
21. *a.* 20 sq. in.
b. 1426 bricks
22. $16.32
23. 15 gal.
24. Ceiling, $6.05; walls, $17.16
25. $48.88
26. *a.* $10.40
b. $29.90
27. *b.* 6 strips 8 ft. long.
c. $117.00
28. 56 yd.
29. 17⅓ yd.; $14.74
30. $281.25

Exercise 77. Pages 151–153

3. *a.* 10.4 in.
b. $16\frac{1}{12}$ ft.
c. $12\frac{9}{16}$ in.
d. 37.3 ft.
e. 22⅛ in.
f. $23\frac{7}{12}$ ft.
4. *a.* 15 in.
b. 33⅓%
c. 40%
d. 26⅔%
6. *b.* 2⅜ in.
c. 2⅝ in.
d. $3\frac{1}{16}$ in.
e. They cross at one point.
8. *a.* 3
b. 1
c. 3
9. *b.* 3¾ in.
c. 3¼ in.
d. 3¾ in.

Exercise 78. Pages 154–155

2. *b.* 3¾ in.
c. 9 in.
d. 53°; 37°
3. *b.* $4\frac{3}{16}$ in.
c. $10\frac{1}{16}$ in.
4. 50 ft.
5. *b.* XZ = about 46½ in.
Buy 52½ in.
c. $13.77
6. b. $5\frac{11}{16}$ ft.

Exercise 79. Pages 155–156

1. *e.* 6 sq. in.
f. 3 sq. in.
2. 20 sq. in.; 10 sq. in.
3. 12 sq. in.
4. 20 sq. ft.
5. 15 sq. in.
6. 30 sq. ft.
7. 22 sq. in.
8. 12.6875 or 12.7 sq. ft.
9. 257.6875 or 257.7 sq. ft.
10. 801.96 sq. ft.

Exercise 80. Pages 156–157

1. *d.* 24 sq. in.; 12 sq. in.
2. 8 sq. in.; 4 sq. in.
3. 18 sq. in.; 9 sq. in.
4. *a.* 20 sq. in.
b. 14 sq. ft.
c. 10.5 sq. yd.

Exercise 81. Pages 158–159

1. 47.25 sq. in.
2. 7.3125 sq. ft.
3. 60 sq. rd.
4. $37\frac{7}{9}$ sq. yd.
5. 192.2 sq. ft.
6. *b.* $3\frac{7}{16}$ in.
c. 7.8 sq. in.
7. *b.* Alt., $2\frac{3}{8}$ in.
Area, 4.45 sq. in.
c. Alt., 3.5 in.
Area, 4.375 sq. in.
8. *a.* 78 sq. ft.
b. Back,, 80 sq. ft.
Front, 128 sq. ft.
c. \$10.92
9. *b.* $4\frac{3}{8}$ in.
c. $1\frac{5}{8}$ in.; 3.5 sq. in.
d. $3\frac{1}{4}$ in.; 7 sq. in.
e. 10.5 sq. in.
10. *a.* 900 sq. ft.
b. 4800 sq. ft.
c. \$148.33$\frac{1}{3}$

Exercise 82. Pages 160–161

2. *c.* Sum = 180°
3. 50°
4. *a.* 70°
b. 78°
c. 85°
5. 90°
6. Acute angles
7. *a.* $\angle C = 60°$
b. $66\frac{2}{3}\%$
c. 200%
d. $33\frac{1}{3}\%$
8. *a.* 80°
b. $133\frac{1}{3}°$
c. 75%; 50%
d. 150%; 200%
9. 50°
10. 60%

Exercise 83. Pages 161–162

3. *b.* 2.25 in.
c. $\angle R = 54°$
$\angle S = 43°$

Exercise 84. Pages 162–164

2. *a.* .....
b. $\angle X = 48°$
$\angle Z = 48°$
3. *b.* $\angle H = \angle K$
5. *d.* Isosceles triangle
6. *b.* 33°
c. About 18 ft.
7. *c.* Each is 60°
8. *b.* Each is 60°
9. 60°
10. *b.* 8.625 in.
c. 2.5 in. (about)
d. All equal.

Exercise 85. Pages 164–166

1. AB = CD; AD = BC
2. Parallel; parallel.
3. *a.* $\angle A = \angle C$;
$\angle B = \angle D$
b. $\angle A + \angle B = 180$
5. *d.* DE = CF
6. *f.* AE = 1¾ in. (about)
Area ADFE = 7 sq. in. (about)
g. 7 sq. in.
7. 12 sq. in.
8. 48 sq. in.
9. 48 sq. yd.
10. 175 sq. ft.
11. 100 sq. ft.
12. 150 sq. in.

Exercise 86. Page 167

1. 546 sq. in.
2. 88 sq. rd.
3. 502½ sq. ft.
4. 19⅛ sq. in.
5. 209.25 sq. in.
6. 409.2 sq. rd.
7. 117 sq. ft.
8. 12.5 sq. yd.
9. 77$\frac{7}{12}$ sq. ft.
10. 35$\frac{5}{9}$ sq. yd.
11. 408⅓ sq. ft.

Exercise 87. Pages 167–169

1. *a.* Parallel
b. No
c. $\angle A + \angle B = 180°$
2. *d.* Trapezoid
e. $\angle D + \angle C = 180°$
3. *c.* AF = EB
c. $\angle A = \angle B$;
$\angle E = \angle F$
4. *b.* Each = 3⅛ in.

Exercise 88. Pages 169–170

1. *d.* Parallelogram
e. ABCD = BFGC
f. *8 sq. in.*
g. *4 sq. in.*
2. *a.* 16 sq. in.
b. 8 sq. in.
3. Parallelogram, 20 sq. in.
Trapezoid, 10 sq. in.
4. 10½ sq. in.
5. 13½ sq. in.

Exercise 89. Pages 170–171

1. 20 sq. in.
2. 80 sq. ft.
3. 34 sq. yd.
4. 30 sq. ft.
5. 60 sq. ft.
6. $21\frac{1}{2}$ sq. in.
7. 1.9439 sq. in.
8. 53.53 sq. in.
9. 16.125 sq. ft.
10. 55 sq. ft.
11. 112 sq. ft.
12. 198 sq. in.
13. 148.5 sq. in
14. 786 sq. in.

Exercise 90. Page 172

7. 141.5 rd.
8. .....
9. 961 sq. ft.
10. 40.96 sq. in.
11. $69.38
12. $4\,1\frac{1}{16}$ a.
13. $5.70

Exercise 91. Page 173

2. 11 in.
3. 15 sq. ft.
4. $2\frac{3}{4}$ in.
8. 37.375 sq. in.
9. *b.* BC = $3\frac{9}{16}$ in.
perimeter, $10\frac{9}{16}$ in.
c. Alt., $2\frac{5}{16}$ in.
10. $4\frac{3}{4}$ in.

Exercise 92. Pages 173–174

1. Perimeter, $7\frac{1}{4}$ in.
2. Perimeter, 8.25 in.
3. Perimeter, 5.5 in.
4. Third angle, 80°.

Exercise 95. Pages 176–177

1. *a.* $\frac{1}{2}$
b. 2 times
4. Diameter
5. They are equal

Exercise 96. Page 179

1. *a.* 3.136; 3.15; 3.13; 3.16.
b. 3.14+

Exercise 97. Pages 180–182

2. 3.1428
3. 18.8 in.
4. 31.416 in.
5. 47.1 ft.
6. 26.7036 or 26.7 in.
7. 7.3 yd.
8. 16.4934 or 16.5 ft.
9. *51.85 ft.*
10. *11.781 ft.*
11. 80.14 in.
12. 104.72 in.
13. 47.1 in.
14. 70.874 ft.
15. 16.34 yd.
16. 74.1 in.
17. 82.72 ft.
18. 29.0598 or 29.1 ft.
19. 115.5 in.

20. 99.463 in.
21. 103.7 in.; 81.7 in.
22. 672.2 times or 672 times.
23. 50.3 in.
24. 24,856.3392 or 24,856.3 mi.
25. 39 in.
26. 67.5444 or 67.5 in.
27. *a.* 9
b. 6.5
c. 15
28. *a.* 4.13 in.
b. 59.6 ft.
c. 15.9 in.
29. *a.* 420 ft.
b. 1477 ft.
c. 1398$\frac{4}{7}$ ft.
30. *a.* 56.5 in.
b. 15.7 in.
c. 3.6 times

Exercise 98. Pages 183–185

1. 201.14 sq. in.
2. 314.16 sq. in.
3. 9.62 sq. in.
4. 452.3904 or 452.4 sq. in.
5. 1964.28 or 1964.3 sq. in.
6. 120.763 or 120.8 sq. in.
7. *a.* Second is 4 times first.
b. Second is 2 times first.
8. Square exceeds circle by 13.7344 or 13.7 sq. in.
9. 11.04 sq. in.
10. *a.* 176.715 or 176.7 sq. in.
b. 19,437 lb.
11. 3$\frac{1}{7}$ or 3.14 sq. mi.
12. 218.16 sq. yd.
13. 113$\frac{1}{7}$ or 113.14 sq. in.
14. *a.* 95.0334 or 95 sq. ft.
b. 380 tulips.
15. $1.51
16. 616 sq. in.
17. 52$\frac{5}{21}$ or 53 plants.
18. 1175 ft.
19. 85¢
20. *a.* 1074.4 ft.
b. 28,824.18 or 28,824 sq. ft.
c. 61,779.564 or 61,780 sq. ft.
d. $22,309.44
e. $6053.25

Exercise 99. Page 185

1. 16	**4.** 100	**7.** 196	**10.** 42.25	**13.** 5.0625
2. 36	**5.** 144	**8.** 225	**11.** 10.24	**14.** 16.5649
3. 64	**6.** 121	**9.** 784	**12.** 53.29	**15.** 29.4849

Exercise 100. Page 186

1. 201.062 or 201 sq. in.
2. 314.16 or 314.2 sq. in.
3. 380.1336 or 380.1 sq. in.
4. 615.7536 or 615.8 sq. in.
5. 530.9304 or 530.9 sq. in.
6. *706.86* or 706.9 sq. ft.
7. *153.9384* or 153.9
8. 254.4696 or 254.5
9. 176.715 or 176.7
10. 226.9806 or 227
11. 122.7
12. 283.5294 or 283.5
13. 89.361 or 89.4
14. 346.3614 or 346.4

Exercise 101. Pages 187–188

1. *e.* AE = EB
   f. Right angles.
5. *a.* All three meet at one point.
   b. The distances are equal.
7. OA = OB
   The distances to A and B are equal.

Exercise 102. Pages 188–189

1. *e.* ∠AOE = ∠EOB
   Bisector of the angle.
4. They meet at one point.

Exercise 103. Pages 189–190

1. *e.* Right angles.
4. They meet at one point.

Exercise 104. Page 190

1. *d.* Right angles.
   Perpendicular lines.
4. Parallel lines.

Exercise 105. Pages 191–194

2. *a.* The arcs are equal.
   b. Six
   c. The chords are equal.
3. *a.* 4; right angles.
   c. Square
6. *a.* The triangle formed has equal sides and equal angles.

Exercise 106. Pages 194–195

5. 28.274 or 28.3 in.
6. 23.562 or 23.6 in.
11. 26.7036 or 26.7 in.
    21.04872 or 21.05 in.
12. 153.938 or 153.9 sq. ft.
16. 64; 49; 121; 26.01
18. 145.267584 or 145.3
19. *55.417824 or 55.4*
20. *36.1284 or 36.1*

Exercise 107. Page 195

5. $1\frac{11}{16}$ in.

Exercise 108. Pages 196–197

2. *a.* Square
   b. 4 in.
3. 8
4. 27
5. 216
6. 125
7. 512
8. 1000
9. 15.625
10. 132.651
12. 1728 cu. in.
13. 1687.5 lb.
14. 4471.875 lb.
15. 156.25 lb.
16. 450.625 lb.
17. 15 lb.

Exercise 109. Page 198

1. *a.* No; three; four.
   b. No; yes.
3. *a.* 12
   b. 2
   c. 24
4. 30

Exercise 110. Pages 199–201

1. 72 cu. ft.
2. 112 cu. yd.
3. 726 cu. ft.
4. $27\frac{7}{9}$ cu. ft.
5. $137\frac{7}{9}$ cu. yd.
6. 4032 cu. ft.
7. $112.00
8. *a.* 20 cu. ft.
   b. 150 gal.
   c. 9375 lb.
9. $66\frac{2}{3}$ cu. ft.
10. 14.8 cu. yd.
11. 23.68 or 23.7 t.
12. $11\frac{1}{19}$ t.
13. 10.8 t.
14. 320 bu.
15. 5.78 or about $5\frac{3}{4}$ ft.
16. *a.* 400 bu.
    b. $2\frac{2}{5}$ ft.
17. 9900 cu. ft.
18. 128 cu. ft.
19. 45 cords.
20. 46.8 lb.
21. 155+ lb.
22. 26,250
23. 131.57 t.
24. *a.* $2933\frac{1}{3}$ cu. yd.
    b. 32.6 or 33 d.
    c. $4207.50

Exercise 111. Pages 201–210

1. $2813.55
2. 1.375
3. .562
4. 42 cu. in.
5. *a.* 5,357,810 bu.
   b. 845,970 bu.
6. Int., $23.75; amount, $973.75.
7. *Int.*, $11.25; amount, $761.25.
8. *$33\frac{1}{3}$%*
9. *$112.50*
10. $285.17
11. *a.* $7.50
    b. $7.50
12. $16\frac{2}{3}$%
13. 3100
14. Work shirt, $27\frac{1}{2}$%
    Man's hat, $13\frac{1}{3}$%
    Overalls, $22\frac{2}{9}$%
    Muslin, 38.08%
    Misses' coat, 20%

15. $12.38
16. $15.30
17. 1203.75 lb
18. *a.* $15.72
b. $18.08
c. $17.18
d. 9.2%
19. *a.* 400%
b. 80%
20. $14.90
21. *a.* $1.00
b. $6.00; $72.00
c. 48%
d. 75¢
22. *a.* 14.1%
b. 129,805,000
23. *b.* $2537.50
24. *a.* Commission, $327.50
Total cost, $3602.50
b. $4103.75
25. *a.* 12,800
b. 16,000
c. 5120
d. 3840
e. 1920
f. 1706⅔
g. 3200
h. 1024
i. 768
j. 731¾
k. 7680
26. *a.* 16⅔%
b. $112.50
27. *a.* 15%
b. 10%
c. Failed, 8⅓%
poor, 25%
fair, 41⅔%
good, 16⅔%
excellent, 8⅓%
28. *a.* $2.00
b. 20%
c. 20¢
d. 12½%
e. 10%
29. *a.* 16⅔%
b. Net profit, 18¢;
30. $10.42
31. *a.* $68
b. $8
c. $5 profit
32. $498.75
33. *a.* 50%
b. 28⅔%
c. 37½%
34. $25.55
35. *a.* $159.38
b. $6896.88
36. 500%
37. *a.* $198.00
b. $19.80
c. 66⅔%; $13.20
d. 38⅓%; $38.50
e. 23.1%; $18.30
f. $70.00; 35.3%
38. *a.* $28.50
b. 200 pots
c. $6.75
d. $2.81
e. $3.38
f. $41.44
g. $103.60; 52¢
39. *a.* 6⅔ yd.; $16.67
b. 10.5 yd.; $1.26
c. 6⅔ yd.; $1.20
d. $19.13

Exercise 112. Pages 211–212

	I	II	III	IV	V	VI	VII	VIII	IX	X
1.	72	55	75	67	123	64	55	180	140	89
2.	90	88	124	82	149	88	107	217	229	167
3.	137	107	178	114	245	131	132	245	295	252
4.	232	169	231	150	279	198	200	257	340	331
5.	306	254	277	207	342	256	279	349	376	355
6.	372	318	359	276	419	339	352	404	424	450
7.	445	374	424	348	503	438	439	487	453	486
8.	503	446	463	391	602	463	513	533	516	523
9.	529	540	550	426	654	511	575	572	592	547

Exercise 113. Page 212

	I	II	III	IV	V	VI	VII	VIII
1.	657	1600	1543	1318	365.1	71.95	21.232	266.89
2.	1014	1872	1885	1964	954.1	125.35	84.482	1249.36
3.	1649	2252	2341	2339	984.7	198.25	89.309	1256.88
4.	1933	2846	2578	3204	1050	202.81	94.943	2019.97
5.	2406	3655	3440	3671	1757	1053.81	97.793	2096.86
6.	3304	4424	4399	4104	2317.7	1059.59	100.85	2116.12
7.	3773	4810	4888	4372	2400.7	1098.01	129.28	2663.96
8.	4144	5373	5113	5124	2474.5	2039.21	201.13	3339.88
9.	4629	6000	5893	6048	2524.1	2395.49	219.165	3427.51

Exercise 114. Pages 213–214

	I	II	III	IV	V	VI
1.	5,984	994	6,055	9.112	.026	8.794
2.	10,281	205	5,716	2.199	.705	14.123
3.	1,991	4,832	6,079	55.712	4.754	28.891
4.	8,698	5,213	4,517	24.613	11.044	17.003
5.	21,266	8,382	9,510	33.88	7.517	8.116
6.	12,137	10,795	15,118	153.561	13.256	60.087
7.	14,618	10,707	16,232	23.459	11.182	32.987
8.	3,065	11,555	11,362	154.401	7.315	68.723
9.	*2,116*	6,343	1,019	74.911	11.058	29.853
10.	*5,181*	17,898	12,381	229.312	18.433	98.576
11.	*44,092*	29,222	35,224	267.766	36.571	114.087

Exercise 115. Pages 214–215

		I	II	III	IV	V	VI	VII
1.	*a.*	45	63	129	7.8	1.08	.858	144.753
	b.	75	105	215	13	1.8	1.43	241.255
	c.	90	126	258	15.6	2.16	1.716	289.506
	d.	105	147	301	18.2	2.52	2.002	337.757
	e.	1.20	1.68	3.44	.208	.0288	.02288	3.86008
	f.	13.5	18.9	38.7	2.34	.324	.2574	43.4259
2.		675	945	1,935	117	16.20	12.87	2171.295
4.		1,596	2,736	4,503	48.45	2,969.7	185.25	3866.994
5.		7,565.8	20,851.2	31,023.4	2,258.88	1,605.884	1,708.925	3241.3842
6.		99,450	248,352	414,726	33,766.2	58,718.4	43,989.66	5946.642
7.		457,107.2	545,179.8	589,736.4	72,141.96	8,191.414	29,719.536	8184.5082
8.		205,239	252,784	292,115	30,502.33	27,447.34	24,217.129	3231.876
9.		32,433.1	34,745.9	39,494	4,630.843	4,273.535	3,743.4726	78.4392
10.		54,366.8	59,588.4	66,918.8	238.408	250.104	406.912	673.03

Exercise 116. Pages 215–216

		I	II	III	IV	V	VI	VII
1.	*a.*	3.00	4.20	8.60	.52	.07	.05	9.65
	b.	2.50	3.50	7.16	.43	.06	.04	8.04
	c.	18.75	26.25	53.75	3.25	.45	.35	60.31
	d.	2.14	3.00	6.14	.37	.05	.40	6.89
	e.	16.66	23.33	47.77	2.88	.40	.31	53.61
2.		1.21	2.08	3.43	.03	2.26	.14	2.94
3.		5.57	15.62	22.85	1.66	1.18	1.25	2.38
4.		21.83	54.52	91.04	7.41	12.89	9.65	1.30
5.		182.47	217.63	235.42	28.79	3.27	1.18	3.26
6.		73.96	91.09	105.26	11.02	9.89	8.72	1.16
7.		97.33	104.27	118.52	13.89	12.82	11.23	.23
8.		163.16	178.83	200.83	.71	.75	1.22	2.01

Exercise 117. Page 216

1.	$\frac{5}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{3}{2}$
2.	$1\frac{1}{12}$	$\frac{7}{12}$	$\frac{1}{8}$	$4\frac{1}{2}$
3.	$1\frac{7}{20}$	$\frac{3}{20}$	$\frac{9}{20}$	$1\frac{1}{4}$
4.	$1\frac{9}{40}$	$\frac{1}{40}$	$\frac{3}{8}$	$2\frac{4}{25}$
5.	$1\frac{1}{2}$	$\frac{1}{6}$	$\frac{5}{6}$	$\frac{4}{5}$
6.	$1\frac{1}{24}$	$\frac{7}{24}$	$\frac{1}{4}$	$\frac{9}{16}$
7.	$1\frac{5}{8}$	$\frac{1}{8}$	$\frac{21}{32}$	$1\frac{1}{6}$
8.	$1\frac{5}{24}$	$\frac{11}{24}$	$\frac{5}{16}$	$2\frac{3}{4}$

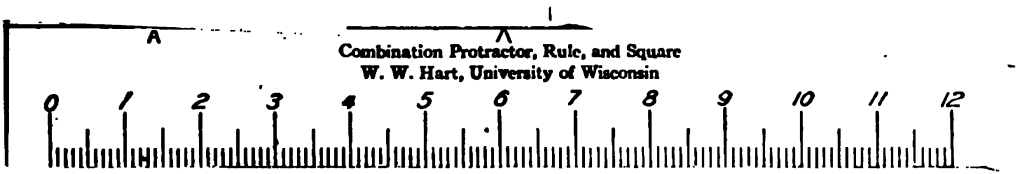
9.	$1\frac{13}{30}$	$\frac{7}{30}$	$\frac{1}{2}$	$\frac{18}{25}$
10.	$1\frac{13}{24}$	$\frac{5}{24}$	$\frac{7}{12}$	$\frac{16}{21}$
11.	$1\frac{1}{16}$	$\frac{7}{16}$	$\frac{15}{64}$	$\frac{5}{12}$
12.	$1\frac{7}{16}$	$\frac{3}{16}$	$\frac{65}{128}$	$1\frac{3}{10}$
13.	$\frac{23}{30}$	$\frac{1}{15}$	$\frac{7}{48}$	$\frac{21}{25}$
14.	$1\frac{43}{48}$	$\frac{37}{48}$	$\frac{3}{4}$	$\frac{27}{64}$
15.	$1\frac{23}{32}$	$1\frac{9}{32}$	$\frac{21}{64}$	$\frac{7}{48}$
16.	$12\frac{1}{6}$	$\frac{5}{6}$	$36\frac{5}{6}$	$1\frac{5}{24}$
17.	$17\frac{7}{8}$	$4\frac{5}{8}$	$14\frac{17}{32}$	$\frac{45}{63}$
18.	$30\frac{5}{6}$	$5\frac{5}{6}$	$229\frac{1}{6}$	$\frac{15}{22}$
19.	125	50	$3281\frac{1}{4}$	$\frac{3}{7}$
20.	100	$33\frac{1}{3}$	$2229\frac{3}{9}$	2
21.	$91\frac{2}{3}$	75	$684\frac{4}{9}$	10
22.	100	25	$2343\frac{3}{4}$	$1\frac{2}{3}$
23.	$14\frac{19}{20}$	$7\frac{9}{20}$	42	$\frac{7}{224}$
24.	$16\frac{7}{24}$	$2\frac{23}{24}$	$64\frac{1}{6}$	$17\frac{1}{160}$
25.	$21\frac{2}{3}$	$11\frac{1}{3}$	$85\frac{1}{4}$	$3\frac{9}{51}$
26.	$14\frac{1}{4}$	$2\frac{7}{8}$	$48\frac{179}{256}$	$\frac{91}{137}$
27.	$48\frac{5}{12}$	$35\frac{1}{12}$	$278\frac{1}{3}$	$6\frac{21}{80}$
28.	$21\frac{1}{4}$	$10\frac{1}{4}$	$86\frac{5}{8}$	$2\frac{19}{22}$
29.	$43\frac{3}{4}$	$18\frac{3}{4}$	$390\frac{5}{8}$	$2\frac{1}{2}$
30.	$264\frac{7}{12}$	$247\frac{11}{12}$	$2135\frac{5}{12}$	$30\frac{3}{4}$

Exercise 118. Pages 216–217

1. 60	14. 840	26. 22	38. 216	50. $\frac{1}{4}$
2. 48	15. 248	27. 60	39. 252	51. $\frac{5}{128}$
3. 54	16. 6.0375	28. 96	40. 2.16	52. $\frac{5}{2}$
4. 45	17. 15.93	29. 256	41. $\frac{1}{6}$	53. $\frac{3}{8}$
5. 99	18. 81	30. 640	42. $\frac{9}{32}$	54. $\frac{11}{36}$
6. 55	19. 1.68	31. 1024	43. $\frac{5}{9}$	55. $\frac{3}{5}$
7. 15	20. 84	32. 55	44. $\frac{7}{10}$	56. $\frac{69}{92}$
8. $37\frac{1}{2}$	21. 14.63	33. .80	45. $\frac{1}{6}$	57. $\frac{1}{30}$
9. 81	22. $307\frac{1}{5}$	34. 2560	46. $\frac{3}{11}$	58. $\frac{3}{7}$
10. 780	23. 80	35. 992	47. $\frac{13}{40}$	59. $\frac{1}{12}$
11. 96	24. 90	36. 49.245	48. $\frac{3}{10}$	60. $\frac{5}{8}$
12. 2.97	25. 96	37. 53.1	49. $\frac{1}{3}$	61. $\frac{3}{4}$
13. .3				

Exercise 119. Pages 218–219

1. 396
2. 59⅜
3. 6½
4. 20¼
5. 87
6. 7
7. 42
8. 35
9. 12.43
10. 28½
11. 1.43
12. 642
13. 2312
14. 6.57
15. 26.257
16. 130
17. 12.351
18. 43.375
19. 917
20. 4.36875
21. $63.75
22. 1900
23. 216⅔
24. 810
25. 34.8
26. 900
27. 308
28. 441
29. 22.6
30. 11.14⅔
31. 200
32. 8200
33. 820
34. 345.6
35. 6100
36. 330⅔
37. 76.2
38. 284
39. 169¼
40. 119¼
41. 16⅔%
42. 20%
43. 33⅓%
44. 30%
45. 5%
46. 150%
47. 220%
48. 350%
49. 8%
50. 11%
51. 20%
52. 205%
53. 82%
54. 3⅓%
55. 181.18%
56. 25%
57. 275%
58. 20.46%
59. 16⅔%
60. 75%
61. 83⅓%


A
Combination Protractor, Rule, and Square
W. W. Hart, University of Wisconsin
0 1 2 3 4 5 6 7 8 9 10 11 12